Honors PreCalculus '20-21 Name: Solution Key

Dr. Quattrin

Rational Functions Test v1

CALCULATOR ALLOWED Score _____

Round to 3 decimal places. Show all work.

1. The equation of the line normal to the graph of $y=\dfrac{x}{2x-3}$ at the point $(1, f(1))$ is

$f(1)=-1$

$f'(1)=3$

(a) $3x+y=4$ (b) $3x+y=2$ (c) $x-3y=-2$

(d) $x-3y=4$ (e) $x+3y=2$

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2. $\lim\limits_{x\to\infty}\dfrac{4x^5+3x^4+2x^3+x^2+1}{3x^6-9x^4+4x^3+15}=$

a) 0 b) $\frac{3}{4}$ c) $\frac{4}{3}$ d) 3 e) DNE

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3. Let $f(x)$ and $g(x)$ be differentiable functions. The table below gives the values of $f(x)$ and $g(x)$, and their derivatives, at several values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	–5	–1	6	1
1	–5	1	6	–1
2	–3	3	4	–3

Find $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$, when $x=0$.

$$\frac{g(0)\cdot f'(0) - f(0)\cdot g'(0)}{(g(0))^2} = \frac{6(-1)-(-5)(1)}{6^2}$$

a) $\frac{7}{36}$ b) $-\frac{5}{36}$ c) $\frac{5}{36}$ d) $\frac{1}{36}$ e) $-\frac{1}{36}$

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4. Which of the following functions has a slant asymptote with a positive slope?

a) $y=\frac{x^3-5x^2-2x+24}{2x^2+7x-15}$

b) $y=\frac{2x^3-9x^2+7x+6}{6-x-x^2}$ $\approx \frac{2x^3}{-x^2}$

c) $y=\frac{x^3-8}{x^4-13x^2+36}$

d) $y=\frac{x^2-1}{x^2+2x-3}$

e) $y=\frac{1-x^2}{x^2+2x-3}$

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5. Suppose $f'(x)=\dfrac{(x+4)^3(x-2)^2}{(x^4+1)}$. Which of the following statements must be **true**?

~~I.~~ The slope of the line tangent to $y=f(x)$ at $x=1$ is 125. F $y'(1)=\frac{125}{2}$

~~II.~~ $f(x)$ is decreasing on $x\in(-4,\ 2)$ F

III. $f(x)$ has a minimum at $x=-4$ T

$-\ 0\ +\ 0\ +$; -4, 2

a) I only b) II only c) III only

d) II and III only e) I, II and III

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6. If $y=\dfrac{5x-4}{4x-5}$, then $\dfrac{dy}{dx}=$ $\dfrac{(4x-5)(5)-(5x-4)(4)}{(4x-5)^2}=\dfrac{20x-25-20x+16}{(4x-5)^2}$

a) $\dfrac{-9}{(4x-5)^2}$ b) $\dfrac{9}{(4x-5)^2}$ c) $\dfrac{40x-41}{(4x-5)^2}$

d) $\dfrac{40x+41}{(4x-5)^2}$ e) $\dfrac{5}{4}$

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7. The POE(s) of $y=\dfrac{x^2+2x-3}{x^2+5x+6}$ is/are at $\dfrac{(x+3)(x-1)}{(x+3)(x+2)}$

a) $x=1$ b) $x=-3$ c) $x=1\ \&\ -3$

d) $x=-2$ e) $x=-2\ \&\ -3$

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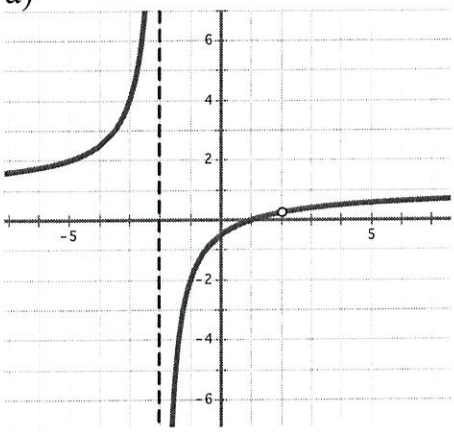
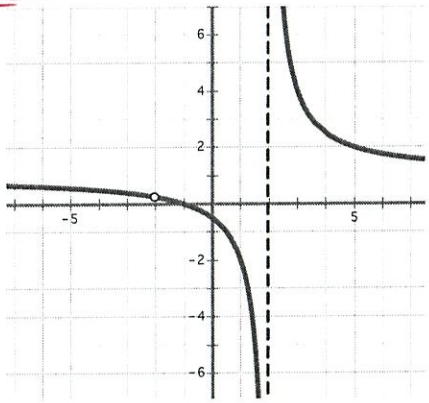
8. Which of the following graphs match the equation $y = \dfrac{x^2+3x+2}{x^2-4}$?

$= \dfrac{(x+2)(x+1)}{(x+2)(x-2)}$

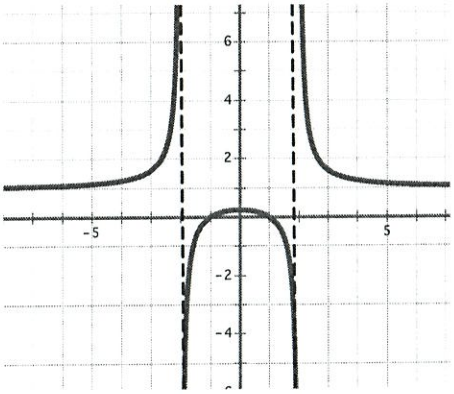
a)



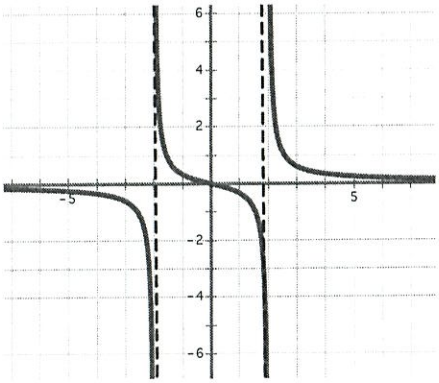
b)



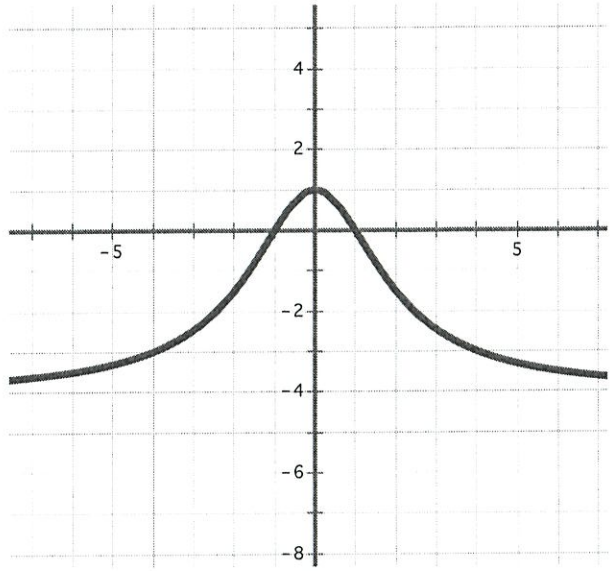
c)



d)



9. Which of the following equations match this graph?



Zeros $(\pm 1, 0)$ so

$y = \frac{k(x^2-1)}{\text{NO VAS}}$

a) $y = \frac{4x}{x^2+4}$

b) $y = \frac{x^2-3x-4}{x^2+4}$

c) $y = \frac{4x^2-4}{x^2+4}$

d) $y = \frac{4-4x^2}{x^2-4}$

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Rational Functions Test v1

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Round to 3 decimal places.

Show all work.

1. Find asymptotes, POEs, and zeros of $y=\dfrac{x^3-5x^2+6x-30}{x^2-8x+15}$. Show the algebraic work to support the zeros.

$x^2(x-5)+6(x-5)$

$\dfrac{(x^2+6)(x-5)}{(x-5)(x-3)}$

ZEROS: NONE

VAs: $x=3$

POE: $\left(5, \frac{31}{2}\right)$

SA: $y=x+3$

$$\begin{array}{r} x+3 \\ x-3\overline{)\,x^2+6} \\ -(x^2-3x) \\ \hline 3x+6 \\ (3x-9) \\ \hline \end{array}$$

2. Apply the First Derivative Test to $y = \dfrac{x^3 - 5x^2 + 6x - 30}{x^2 - 8x + 15}$, and Find the extreme points.

$$\frac{dy}{dx} = \frac{\cancel{(x^2+6)}(2x)}{(x-3)(4) - (x^2+6)(1)}{(x-3)^2}$$

$u = x^2+6$

$du = 2x$

$v = x-3$

$dv = 1$

$$= \frac{2x^2 - 6x - x^2 - 6}{(x-3)^2}$$

$$= \frac{x^2 - 6x - 6}{(x-3)^2}$$

i) $\dfrac{dy}{dx} = 0 \rightarrow x = \dfrac{6 \pm \sqrt{36+24}}{2} = \begin{cases} 6.873 \\ -.873 \end{cases}$

ii) $\dfrac{dy}{dx}$ DNE $\rightarrow x = 3$ NOT IN DOMAIN

iii) ENDPOINTS NONE GIVEN

$\frac{dy}{dx}$	+	0	−	VA	−	0	+
x		$-.873$		3		6.873	
		MAX				MIN	

3. $\frac{d}{dx}\left[\frac{x^2-5x+6}{x^2+6x+8}\right] = \frac{d}{dx}\left[\frac{(x-3)(x-2)}{(x+2)(x+4)}\right]$

$$= \frac{(x^2+6x+8)(2x-5)-(x^2-5x+6)(2x+6)}{(x^2+6x+8)^2}$$

$$= \frac{\cancel{2x^3}+7x^2-14x-40-\cancel{2x^3}+4x^2+18x-36}{(x^2+6x+8)^2}$$

$$= \frac{11x^2+4x-76}{(x^2+6x+8)^2}$$

4. Find the zeros, VAs, POEs and EB of $y=\frac{9-x^2}{x^2-4}$ on $x\in[-1, 5]$. Show the derivative and algebra to support the critical values.

Zeros: $(\cancel{\pm 3}, 0) \rightarrow (3, 0)$

VAs: $x = \pm 2 \rightarrow x = 2$

POE: NONE

End Behavior (left): NONE

End Behavior (right): NONE

5. Find the Extreme Points of $y=\frac{9-x^2}{x^2-4}$ on $x\in[-1,\,5]$. Show the derivative and algebra to support the critical values.

$$\frac{dy}{dx}=\frac{(x^2-4)(-2x)-(9-x^2)(2x)}{(x^2-4)^2}$$

$$=\frac{-2x^3+8x-18x+2x^3}{(x^2-4)^2}=\frac{-10x}{(x^2-4)^2}$$

i) $\frac{dy}{dx}=0 \rightarrow x=0 \rightarrow (0,\,-9/4)$

ii) $\frac{dy}{dx}=\text{DNE} \rightarrow x^2+4=0 \rightarrow x=\pm 2$, NOT IN DOMAIN

iii) END POINTS $x=-1,\,5 \rightarrow (-1,\,-8/3)$

$(5,\,\frac{-16}{21})$

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NO CALCULATOR ALLOWED Score _____

Show all work.

6. Write an equation of a rational function that has x–intercepts at (-6, 0), VA at $x = 6$, a POE at $x = 3$, and a HA at $y = \frac{9}{4}$.

$$y = \frac{9(x+6)(x-3)}{4(x-6)(x-3)}$$

7. Show the sign pattern and solve $\frac{x^2-5x+6}{x^2+6x+8} \le 0$.

$$\frac{(x-3)(x-2)}{(x+2)(x+4)} \le 0$$

y	+	VA	−	VA	+	0	−	0	+
x		-4		-2		2		3	

~~$x \in (-\infty, -4) \cup (-2, 2] \cup [3, \infty)$~~

$x \in (-4, -2) \cup [2, 3]$

8. Find the traits and **sketch** $y = \dfrac{9 - x^2}{x^2 - 4}$ on $x \in [-1, 5]$.

Domain: $x \in [-1, 2) \cup (2, 5]$

Range: $y \in (-\infty, -9/4] \cup \left[\frac{-16}{31}, \infty\right)$

Y – Int: $(0, -9/4)$

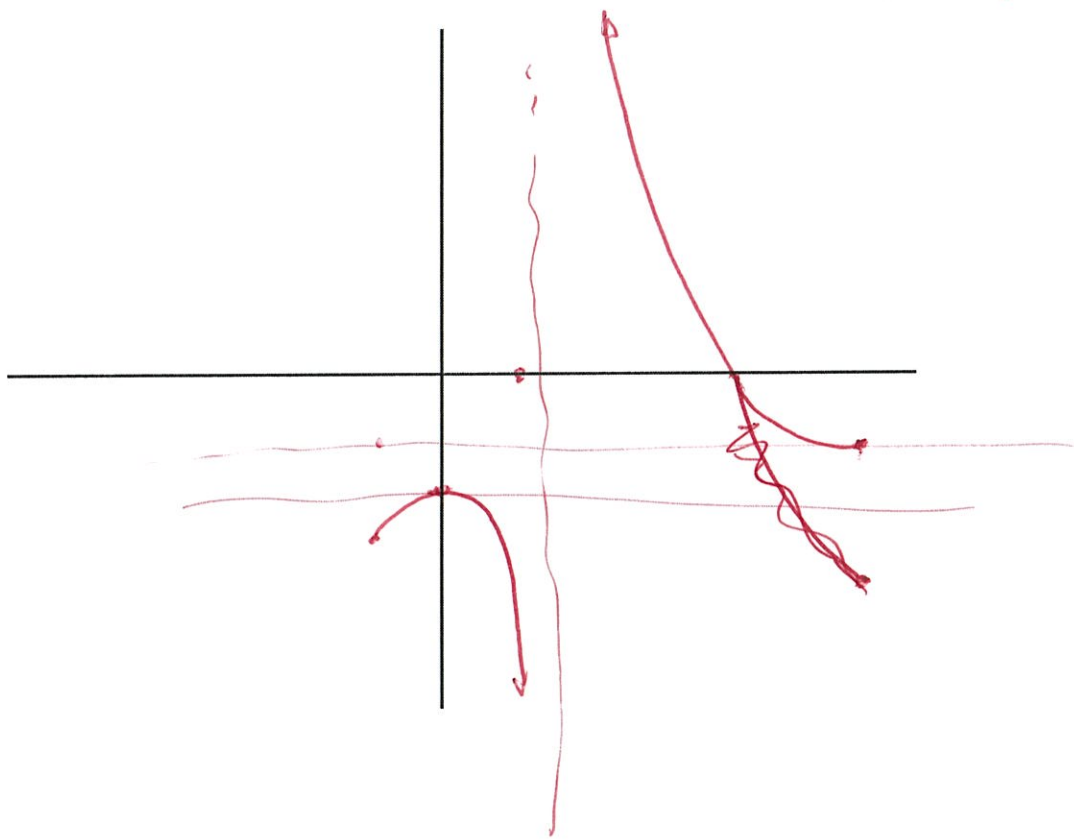
End Behavior: NONE

Vas: $x = 2$

POEs: NONE

Zeros: $(3, 0)$

Extreme Values: $(0, -9/4)$, $(1, -8/3)$, $(5, -16/21)$



9. Find the traits and **sketch** of $y=\dfrac{x^3-5x^2+6x-30}{x^2-8x+15}$.

Domain: $x \neq +3$

Range: $y \in (-\infty, -1.746] \cup [13.746, \infty)$

Y – Int: $(0, -2)$

End Behavior: $y = x+3$

Zeros: NONE

Extreme ~~Values~~ POINTS: $(-.873, -1.746)$ $(6.873, 13.746)$

POEs: $(5, 3\frac{1}{2})$

VAs: $x=3$