Honors PreCalculus '16-17
Trig Identities Group Test
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Name Solutions Key

Score_____

For Problem 1-6, use:

(-7, 24) is on the terminal side of A and
$90° \le A \le 180°$

$\cos B = 4/5$ and $0° \le B \le 90°$; and

$Cot\ C = 12/5$ and $-180° \le C \le -90°$

to find the exact values of:

1. Sin (A-B) $= \sin A \cos B - \cos A \sin B$

$\left(\frac{24}{25}\right)\left(\frac{4}{5}\right) - \left(\frac{-7}{25}\right)\left(\frac{3}{5}\right)$

$\cancel{\frac{72+28}{125}} = \cancel{\frac{4}{5}}\ \frac{117}{125}$

2. $\tan(2C) = \dfrac{2(5/12)}{1-\left(\frac{5}{12}\right)^2}$

$= \frac{5}{6} \cdot \frac{\cancel{240}\ 144}{119} = \frac{120}{119}$

3. $\cos\left(\frac{1}{2}A\right) = +\sqrt{\frac{1}{2}\left(1+\frac{-7}{25}\right)}$

$= \sqrt{\frac{9}{25}} = \frac{3}{5}$

4. Csc (A+B) $= \dfrac{1}{\sin A \cos B + \cos A \sin B}$

$= \dfrac{1}{\frac{24}{25}\left(\frac{4}{5}\right) + \left(\frac{-7}{25}\right)\left(\frac{4}{5}\right)}$

$= \dfrac{125}{96-21} = \dfrac{5}{3}$

5. $\sec(2B) = \dfrac{1}{\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2} = \dfrac{25}{7}$

6. $\tan\left(\frac{1}{2}C\right) = \dfrac{\sin C}{1+\cos C} = \dfrac{-5/13}{1+\frac{-12}{13}}$

$= -5$

7. Prove: $\dfrac{\tan^2 w - \sec w - 5}{\tan^2 w + 3\sec w + 3} = \dfrac{\sec w - 3}{\sec w + 1}$

$\dfrac{\sec^2 w - 1 - \sec w - 5}{\sec^2 w - 1 + 3\sec w + 3}$

$\dfrac{\sec^2 w - \sec w - 6}{\sec^2 w + 3\sec w + 2}$

$\dfrac{(\sec w - 3)\cancel{(\sec w + 2)}}{(\sec w + 1)\cancel{(\sec w + 2)}}$

9. Solve exactly for $x \in [0°, 360°)$:
$\sin\theta = \sin 2\theta$

$0 = 2\sin\theta\cos\theta - \sin\theta$

$= \sin\theta(2\cos\theta - 1)$

$\sin\theta = 0 \qquad \cos\theta = 1/2$

$\theta = 0 \pm 180n \qquad \theta = \pm 60° \pm 360n$

$\theta = \{0, 180, 60, 300\}$

8. Prove: $\cos\left(\dfrac{5\pi}{4} - x\right) = -\dfrac{\sqrt{2}}{2}(\cos x + \sin x)$

$\cos\frac{5\pi}{4}\cos x + \sin\frac{5\pi}{4}\sin x$

$\frac{-1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$

$-\frac{1}{\sqrt{2}}(\cos x + \sin x)$

10. Solve exactly for x:
$2\left(\cot\dfrac{1}{2}x\right)(1 - \cos x)\dfrac{\csc x}{\sec x} = \sqrt{3}$

$2\left(\dfrac{\cancel{\sin x}}{\cancel{1 - \cos x}}\right)\left(\cancel{1 - \cos x}\right)\left(\dfrac{1}{\cancel{\sin x}}\cdot\dfrac{\cos x}{1}\right)$

$\cos x = \dfrac{\sqrt{3}}{2}$

$x = \pm\dfrac{\pi}{6} \pm 2\pi n$

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to find the exact values of:

1. cos (A-B) $= \left(\frac{-7}{25}\right)\left(\frac{4}{5}\right) + \left(\frac{24}{25}\right)\left(\frac{3}{5}\right)$

$= \frac{44}{125}$

2. Sin 2B $= 2\left(\frac{4}{5}\right)\left(\frac{+3}{5}\right)$

$= \frac{24}{25}$

3. $\sec\left(\frac{1}{2}B\right) = \frac{1}{+\sqrt{\frac{1}{2}\left(1+\frac{4}{5}\right)}}$

$= \frac{\sqrt{10}}{3}$

4. tan (A+B) $= \dfrac{\frac{-24}{7} + 3/4}{1 - \left(\frac{-24}{7}\right)(3/4)}$

$= \frac{-75}{100} = \frac{-3}{4}$

5. Cot 2A $= \dfrac{1 - \left(\frac{-24}{7}\right)^2}{2(-24/7)}$

$= \frac{-527}{49} \cdot \frac{7}{-48} = \frac{527}{336}$

6. $\csc\left(\frac{1}{2}C\right) = \frac{1}{\sqrt{\frac{1}{2}\left(1 - \frac{12}{13}\right)}} = -\frac{\sqrt{26}}{5}$

7. Prove: $\cos\left(x+\frac{\pi}{2}\right)\sin(x+\pi)=1-\cos^2 x$

$\left(\cos x \overset{0}{\cos\frac{\pi}{2}} - \overset{1}{\sin x \sin\frac{\pi}{2}}\right) \cdot$

$\left(\sin x \underset{-1}{\cos\pi} + \cos x \underset{0}{\sin\pi}\right)$

$(-\sin x)(-\sin x)$

$\sin^2 x$

$1-\cos^2 x$

9. Prove: $\csc(2x)=\frac{\csc x}{2\cos x}$

$\frac{1}{2\sin x\cos x} = \frac{\frac{1}{\sin x}}{2\cos x}$

8. Solve for $x\in\left[\frac{-\pi}{4},\ \frac{\pi}{4}\right)$:

$\tan^4 x-4\tan^2 x+3=0$

$(\tan^2 x-3)(\tan^2 x-1)$

$\tan^2 x=3$ $\qquad$ $\tan^2 x=1$

$\tan x=\pm\sqrt{3}$ $\qquad$ $\tan x=\pm 1$

$x=\pm\frac{\pi}{3}\pm\pi n$ $\qquad$ $x=\pm\frac{\pi}{4}\pm\pi n$

$x=\left\{\frac{-\pi}{4}\right\}$

10. Solve for x: $2\cos^2\left(\frac{1}{2}x\right)-2=2\cos x$

$2\left(\sqrt{\frac{1}{2}(1+\cos x)}\right)^2-2=2\cos x$

$1+\cos x = 2\cos x+2$

$-1 = \cos x$

$1 \cancel{+\cos x}$

$\boxed{x=\pi\pm 2\pi n}$

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to find the exact values of:

1. Tan (A-B) $= \dfrac{\frac{-24}{7} - \frac{3}{4}}{1 + \left(\frac{-24}{7}\right)\left(\frac{3}{4}\right)}$

$= \dfrac{+117}{44}$

2. Cot 2B $= \dfrac{1 - (3/4)^2}{2(3/4)}$

$= +7/24$

3. $\sin\left(\frac{1}{2}B\right) = \dfrac{\cancel{\text{num}}}{\sqrt{\frac{1}{2}(1 - 4/5)}} = \dfrac{1}{\sqrt{10}}$

4. Cos (A+C) $= \left(\frac{-7}{25}\right)\left(\frac{-12}{13}\right) - \left(\frac{24}{25}\right)\left(\frac{-5}{13}\right)$

$= \dfrac{204}{325}$

5. Csc 2C $= \dfrac{1}{2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)}$

$= \dfrac{169}{120}$

6. $\sec\left(\frac{1}{2}A\right) = \dfrac{1}{+\sqrt{\frac{1}{2}\left(1 + \left(\frac{-7}{25}\right)\right)}}$

$= \dfrac{5}{3}$

7. Prove:

$$\frac{\sin 4x}{4(\cos x+\sin x)} = \sin x\cos^2 x - \cos x\sin^2 x$$

$$\frac{2\sin 2x\cos 2x}{4(\cos x+\sin x)}$$

$$\frac{2(2\sin x\cos x)(\cos^2 x-\sin^2 x)}{4(\cos x+\sin x)}$$

$$\sin x\cos x(\cos x-\sin x)$$

$$\sin x\cos^2 x-\cos x\sin^2 x$$

8. Prove: $\sec^2\frac{\theta}{2} = \frac{2\tan\theta}{\tan\theta+\sin\theta}$

$$= \frac{1}{\sqrt{\frac{1}{2}(1+\cos\theta)}}^2 = \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\sin\theta}$$

$$= \frac{2}{1+\cos\theta} = \frac{2\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta+\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta}{\sin\theta(1+\cos\theta)}$$

9. Solve exactly for x:
$\tan x+\tan 2x = 1-\tan x\tan 2x$

$$\frac{\tan x+\tan 2x}{1-\tan x\tan 2x} = 1$$

$$\tan 3x = 1$$

$$3x = \frac{\pi}{4} \pm \pi n$$

$$x = \frac{\pi}{12} \pm \frac{\pi}{3}n$$

10. Solve exactly for $x\in[0,\ 60^\circ]$:

$$\frac{\cos x\cos 20^\circ-\sin x\sin 20^\circ}{\sin x\cos 20^\circ+\cos x\sin 20^\circ} = \sqrt{3}$$

$$\frac{\cos(x+20^\circ)}{\sin(x+20^\circ)} = \sqrt{3}$$

$$\cot(x+20^\circ) = \sqrt{3}$$

$$\tan(x+20^\circ) = \frac{1}{\sqrt{3}}$$

$$x+20^\circ = 30^\circ \pm 180n$$

$$x = 10^\circ \pm 180n$$

$$\boxed{x = 10^\circ}$$

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$Cot\ C = 12/5$ and $-180° \le C \le -90°$

to find the exact values of:

1. Csc (A-B) $= \dfrac{1}{\frac{24}{25}\left(\frac{4}{5}\right) - \left(\frac{-7}{25}\right)\left(\frac{3}{5}\right)}$

$= \dfrac{125}{117}$

2. Sec 2A $\dfrac{1}{\left(\frac{-7}{25}\right)^2 - \left(\frac{24}{25}\right)^2}$

$= \dfrac{-625}{527}$

3. $\cot\left(\frac{1}{2}A\right) = \dfrac{1 - \left(\frac{24}{25}\right)\left(\frac{-7}{25}\right)}{\frac{24}{25}} = \dfrac{32}{24} = \dfrac{3}{4}$

$= \frac{4}{3}$

4. Sin (B+C) $= \frac{3}{5}\left(\frac{-12}{13}\right) + \frac{4}{5}\left(\frac{-5}{13}\right)$

$= \dfrac{-56}{65}$

5. Cos 2C $= \left(\frac{-12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2$

$= \dfrac{119}{169}$

6. $\sin\left(\frac{1}{2}C\right) = -\sqrt{\frac{1}{2}\left(1 - \left(\frac{-12}{13}\right)\right)}$

$= -5/\sqrt{26}$

7. Prove: $\dfrac{2\cos^2\left(\frac{1}{2}x\right)}{\sin x} = \csc x + \cot x$

$$\frac{2\left[\sqrt{\frac{1}{2}(1+\cos x)}\right]^2}{\sin x} = \frac{1}{\sin x} + \frac{\cos x}{\sin x}$$

$$\frac{1+\cos x}{\sin x}$$

9. Prove: $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$

$$\frac{\sin x}{\frac{1}{\sin x}} + \frac{\cos x}{1/\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

8. Solve for θ: $2\csc^2\theta + 5\cot\theta = 0$

$$2(\cot^2\theta + 1) + 5\cot\theta = 0$$

$$2\cot^2\theta + 5\cot\theta + 2 = 0$$

$$(2\cot\theta + 1)(\cot\theta + 2) = 0$$

$$\cot\theta = -1/2 \quad \text{or} \quad \cot\theta = -2$$

$$\boxed{\theta = -1.107 \pm \pi n \quad \text{or} \quad \theta = -.464 \pm \pi n}$$

10. Solve for $\theta \in [0°, 360°]$: $\cos\theta\sin\theta = \dfrac{\sqrt{3}}{4}$

$$2\sin\theta\cos\theta = \frac{\sqrt{3}}{2}$$

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$

$$2\theta = \begin{cases} 60° \pm 360n \\ 120° \pm 360n \end{cases}$$

$$\theta = \begin{cases} 30° \pm 180n \\ 60° \pm 180n \end{cases}$$

$$\theta = \{30°, 60°, 210°, 240°\}$$