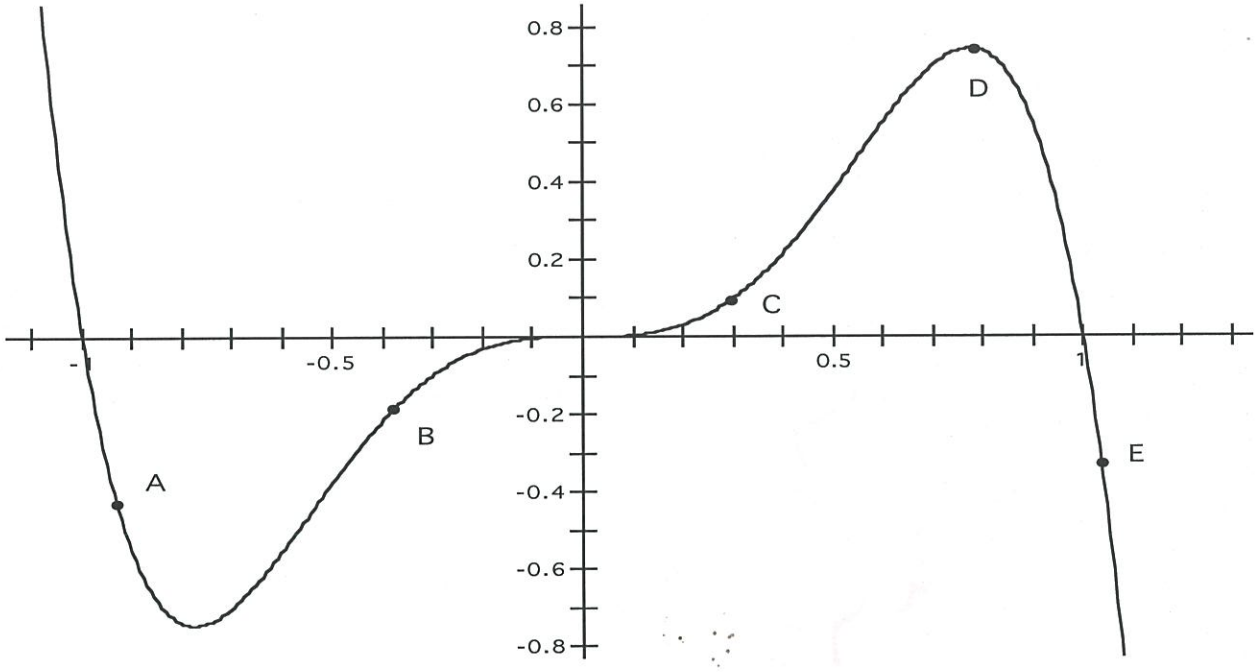
Honors PreCalculus '16-17
Advanced Curve Sketching

Name: SOLUTION KEY

score_____

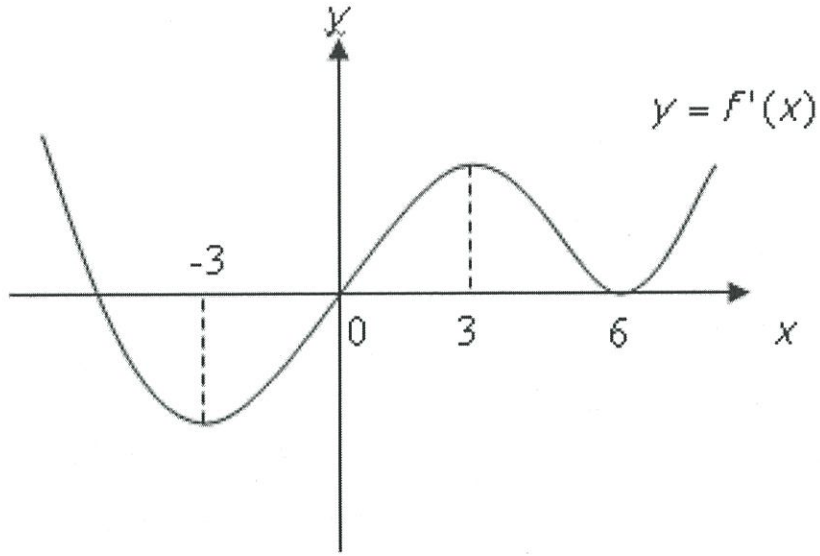


1. The graph of the function $f(x)$ is shown above. At which point on the graph of $f(x)$ is $f'(x) > 0$ and $f''(x) > 0$?

INC CONCAVE UP

(a) A (b) B (c) C (d) D (e) E

2. The graph of the derivative of f is shown below. Which of the following must be true?



a) f has a local maximum at $x = 0$. MIN @ $x = 0$
b) f is increasing on [-3, 3]. f' IS INCREASING
c) f has a point of inflection at $x = 6$. f' SWITCHES FROM DEC TO INC
d) f is concave down on [0, 6].
e) f has a local minimum at $x = -3$.

3. Given this sign pattern $\begin{array}{c} F''(x) \\ x \end{array} \begin{array}{ccccccc} - & 0 & + & 0 & - & 0 & - \\ & -1 & & 0 & & 2 & \end{array}$ and that $f'(-1) = f'(1) = f'(2) = 0$, which of the following statements must be true?

I. $F(x)$ has a local maximum at $x = 1$ $f'(1) = 0$ $f''(1) < 0$
II. $F(x)$ has a local minimum at $x = -1$ $f''(-1) = 0$
III. $F(x)$ has a point of inflection at $x = 2$ NO f'' SIGN CHANGE

(a) I only
(b) II only
(c) III only
(d) I and III only
(e) I, II, and III

4. If $f''(x) = x^2(x+1)^2(x-2)$, then the graph of $f(x)$ has inflection points when $x =$

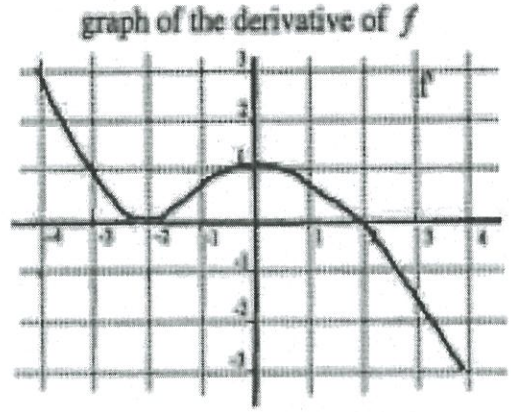
NO SIGN CHANGE @ $x=0$ OR $x=-1$

(a) -1 only
(b) 2 only
(c) -1 and 0 only
(d) -1 and 2 only
(e) -1, 0, and 2 only

5. The graph of the *derivative* of $f(x)$ is shown below. Which of the following is true about the function $f(x)$?

I. $f(x)$ is decreasing at $x=0$
II. $f(x)$ has a local minimum at $x=-2$
III. $f(x)$ is concave up at $x=-1$

(a) I only
(b) II only
(c) III only
(d) II and III only
(e) I, II, and III



f' + 0 + 0 −

f'' − 0 + 0 − 2

−2 0

6. If $f(x) = 7\sin^2(2x)$, then $f''\left(\frac{\pi}{12}\right) =$

(a) 0 (b) 14 (c) $14\sqrt{2}$ (d) 28 (e) $14\sqrt{3}$

$f'(x) = 14\sin(2x)\cos 2x\,(2) = 14\sin 4x$

$f''(x) = 14\cos 4x\,(4) = 56\cos(4x)$

$f''(\pi/12) = 56\cos\left(\frac{\pi}{3}\right) = 28$

Extra Credit

7. $\int (10x + 8x^5)\, dx =$ $\frac{10x^2}{2} + \frac{8x^6}{6} + C$

a) $5x^2 + \frac{4}{3}x^6 + C$ b) $5x^2 + \frac{8}{5}x^5 + C$ c) $10x + \frac{4}{3}x^6 + C$

d) $5x^2 + 8x^6 + C$ e) $5x^2 + \frac{8}{7}x^6 + C$

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8. The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line. Find the velocity at time t ~~and the distance traveled during the given time interval.~~

$a(t) = t + 4,\ v(0) = 3$ $V = \frac{t^2}{2} + 4t + C$ $C = 3$

a) $v(t) = t^2 + 4t + 3$ b) $v(t) = \frac{t^2}{2} + 4t + 3$ c) $v(t) = \frac{t^2}{2} + 3$

d) $v(t) = t^2 + 4t$ e) $v(t) = \frac{t^2}{2} + 4t$

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9. $\frac{1}{3}\int 3x^2\sqrt{x^3+4}\, dx$ $= \frac{1}{3}\frac{u^{3/2}}{3/2} = \frac{2}{9}(x^3+4)^{3/2} + C$

a. $\frac{2}{3}(x^3+4)^{3/2} + c$ b. $\frac{4}{9}(x^3+4)^{3/2} + c$ c. $\frac{2}{9}x^3(x^3+4)^{3/2} + c$

d. $\frac{1}{3\sqrt{x^3+4}} + c$ e. $\frac{2}{3\sqrt{x^3+4}} + c$

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Round to 3 decimal places.
Show all work.

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1. Given $y=\frac{8x}{x^2+4}$, find the sign pattern for $\frac{dy}{dx}$ and determine if the critical values are at a maximum, minimum, or neither.

$$\frac{dy}{dx}=\frac{(x^2+4)(8)-8x(2x)}{(x^2+4)^2}$$

$$=\frac{-8x^2+32}{(x^2+4)^2}$$

$$\frac{dy}{dx}=0 \rightarrow x=\pm 2$$

y': $-$ 0 $+$ 0 $-$

-2 MIN

2 MAX

2. Given $y=\frac{8x}{x^2+4}$, find the sign pattern for $\frac{d^2y}{dx^2}$ and name the points of Inflection.

$$\frac{dy}{dx} = \frac{-8x^2+32}{(x^2+4)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x^2+4)^2(-16x) - (-8x^2+32)\,2(x^2+4)^1(2x)}{(x^2+4)^4}$$

$$= \frac{-4x(x^2+4)^1\left[4(x^2+4) + (-8x^2+32)\right]}{(x^2+4)^4}$$

$$= \frac{-4x(-4x^2+48)}{(x^2+4)^3}$$

$$\frac{d^2y}{dx^2} = 0 \rightarrow x = 0,\ \pm\sqrt{12} = \pm 2\sqrt{3}$$

POIs. $(0, 0)$

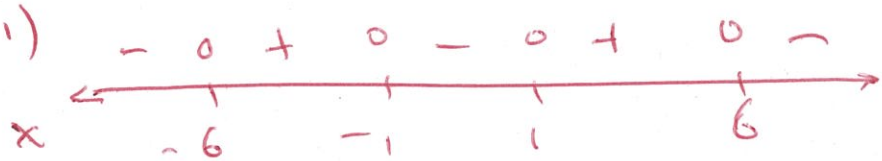
$(2\sqrt{3}, \sqrt{3})$

$(-2\sqrt{3}, -\sqrt{3})$

3. Find the domain, zeros and VAs of $y=\ln\left(-x^4+37x^2-36\right)$.

$-x^4+37x^2-36$

$-(x^2-36)(x^2-1)$



VAs: $x=\pm 1, \pm 6$

Domain $x \in (-6,-1) \cup (1,6)$

Zeros: $-x^4+37x^2-36=1$

$x=\pm 5.998, \pm 1.014$

$(\pm 5.998, 0), (\pm 1.014, 0)$

4. Find the extreme points and end behavior of $y=\ln\left(-x^4+37x^2-36\right)$.

$$\frac{dy}{dx}=\frac{-4x^3+74x}{-x^4+37x^2-36}=0 \rightarrow x=\cancel{0}, \pm\sqrt{37/2}=\pm 4.301$$

$(\pm 4.301, 5.724)$

EB: NONE ON EITHER END BECAUSE OF THE DOMAIN

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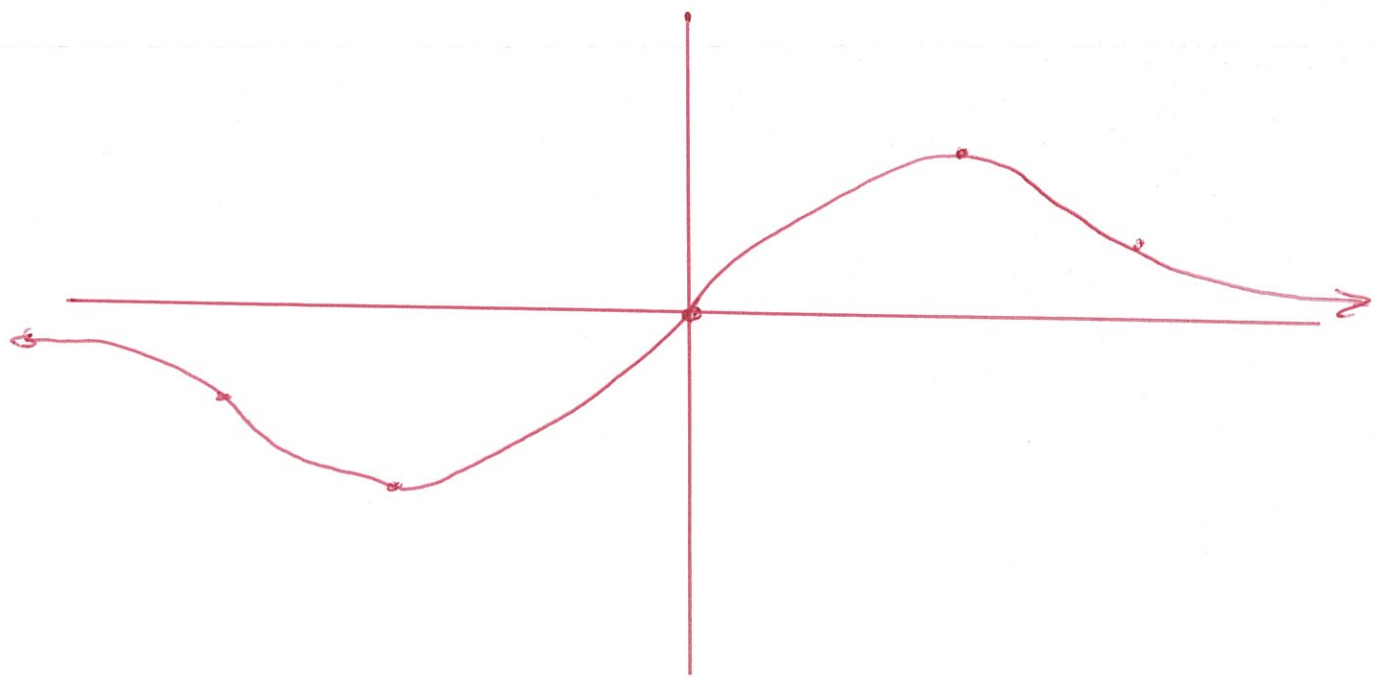
NO CALCULATOR ALLOWED

Show all work.

5. Set up a Key Trait table and sketch $y = \dfrac{8x}{x^2+4}$

x		$-2\sqrt{3}$		-2		0		2		$2\sqrt{3}$	
y		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$	
y'	$-$	$-$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$
y''	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
CONCLUSION	⌉	POI	⌊	MIN	⌋	POI	⌈	MAX	⌉	POI	⌊



6. Show the Algebraic Traits and sketch $y = \ln\left(-x^4 + 37x^2 - 36\right)$.

Domain: $x \in (-6,-1) \cup (1,6)$

Range: $y \in (-\infty, 5.724]$

Y – Int: NONE

VAs: $x = \pm 1, \pm 6$

Zeros: SEE #3

Extreme Points: SEE #4

End Behavior (Left): NONE

POEs: NONE

End Behavior (Right): NONE