Chapter 8 Odd Solution Manual

Section 8–1 Free Response

1. $y=\sqrt{9-x^2}$

$9-x^2\geq 0$

$(3+x)(3-x)\geq 0$

$x=\pm 3$

y^2 $\quad - \quad 0 \quad + \quad 0 \quad -$

x $\quad -3 \quad 3$

Domain: $x\in[-3,\ 3]$

Zeros: $(3,\ 0),\ (-3,\ 0)$

3. $y=\sqrt{x^2-2x-5}$

$x^2-2x-5\geq 0$

$x=3.449,\ -1.449$

y^2 $\quad + \quad 0 \quad - \quad 0 \quad +$

x $\quad -1.449 \quad 3.449$

Domain: $x\in(-\infty,\ -1.449]\cup[3.449,\ \infty)$

Zeros: $(-1.449,\ 0),\ (3.449,\ 0)$

5. $y=\sqrt{x^3-5x^2+7x-3}$

Factor the radicand using synthetic and/or graphing calculator:

$(x-1)(x-3)(x-1)\geq 0$

y^2 $\quad - \quad 0 \quad - \quad 0 \quad +$

x $\quad 1 \quad 3$

Note that the signs around $x=1$ are the same since that factor appears twice.

Domain: $x\in[3,\ \infty)$

Zeros: $(1,\ 0),\ (3,\ 0)$

7. $y=\sqrt{\dfrac{x}{x^2-4}}$

$$\frac{x}{x^2-4} \geq 0$$

$$\frac{x}{(x+2)(x-2)} \geq 0$$

Zero at $x = 0$, VAs at $x = \pm 2$

y^2	$-$	DNE	$+$	0	$-$	DNE	$+$	
x		-2		0		2		

Domain: $x \in (-2,\ 0] \cup (2,\ \infty)$

Zeros: $(0,\ 0)$

9. $y = \sqrt{-x^4 + 41x^2 - 400}$

$$(-x^2+16)(-x^2+25) = 0$$

$$x = \pm 4,\ \pm 5$$

y	$-$	0	$+$	0	$-$	0	$+$	0	$-$
x		-5		-4		4		5	

Zeros: $x = \pm 4,\ \pm 5$

Domain: $x \in [-5, -4] \cup [4, 5]$

11. $y = \sqrt{-x^3 - x^2 + 9x + 9}$

$$-x^2(x+1) + 9(x+1) = 0$$

$$(9 - x^2)(x+1) = 0$$

$$x = \pm 3,\ -1$$

y	$+$	0	$-$	0	$+$	0	$-$
x		-3		-1		3	

Zeros: $x = \pm 3,\ -1$

Domain: $x \in (-\infty, -3] \cup [-1, 3]$

13. $y=\sqrt{x^2-5x-21}$

$x^2-5x-21$
$(x-7)(x+3)=0$
$x=-3,\ 7$

$$\begin{array}{c} y \\ x \end{array} \xleftrightarrow[-3 \qquad 7]{+ \quad 0 \quad - \quad 0 \quad +}$$

Zeros: $x=-3,\ 7$

Domain: $x\in(-\infty,-3]\cup[7,\infty)$

15. $y=\sqrt{\dfrac{x^2-9}{x^2-4}}$

Zeros: $(\pm 3,\ 0)$;

VAs: $x=\pm 2$;

Domain: $x\in(-\infty,\ -3]\cup(-2,2)\cup[3,\infty)$

17. $y=-\sqrt{\dfrac{6x}{x^2+9}}$

Zeros: $(0,\ 0)$ because the only number that makes the numerator = 0 is $x=0$

VAs: none, because no real numbers make $x^2+9=0$

Domain: $\begin{array}{c} y \\ x \end{array} \xleftrightarrow[0]{- \quad 0 \quad +} x\in[0,\infty)$. NB. The negative outside the radical does not affect the sign pattern of the radicand.

19. $y=\sqrt{\dfrac{x^2+3x}{x^2+2x-3}}=\sqrt{\dfrac{(x+3)x}{(x-1)(x+3)}}$

$x=0$ makes only the numerator equal zero. Therefore, $(0,\ 0)$ is the zero.

$x=-1$ makes only the denominator equal zero. Therefore, $x=-1$ is the vertical asymptote.

$x=-3$ makes both the numerator and the denominator equal zero. So, $x=-3$ is at a POE.

y	$+$ POE	$+$	0	$-$	VA	$+$
x	-3		0		1	

Domain: $x\in(-\infty,\ -3)\cup(-3,0]\cup(1,\infty)$

<u>Section 8–1 Multiple Choice</u>

1. Each part of the function must be considered separately:
For $\sqrt{u}$, $u\geq 0$. For $\sqrt{9-u}$, $u\leq 9$. The area of overlap is the domain of the entire function: $0\leq u\leq 9$
(D) is the correct response

3. $\sqrt{x}-\dfrac{1}{x\sqrt[3]{x}}$ can be rewritten as $x^{1/2}-x^{-1}\left(x^{-1/3}\right)=x^{1/2}-x^{-4/3}$. Therefore, using the Power Rule, $D_x=\dfrac{1}{2}x^{-1/2}+\dfrac{4}{3}x^{-7/3}$.
(B) is the correct response

5. The sign pattern for y^2 means that this is a domain sign pattern for a radical function. Values that are positive and zero determine domain.
(E) is the correct response

Section 8–2: The Chain Rule

Free Response Solutions

1. $\frac{d}{dx}\left[\sqrt{2x+25}\right]$

$= \frac{d}{dx}\left[(2x+25)^{1/2}\right]$

$= \frac{1}{2}(2x+25)^{-1/2} \bullet (2)$

$= \frac{1}{(2x+25)^{1/2}}$ *or* $\frac{1}{\sqrt{2x+25}}$

3. $y = \sqrt{x^2+x+13}$

$y = \left(x^2+x+13\right)^{1/2}$

$y' = \frac{1}{2}\left(x^2+x+13\right)^{-1/2} \bullet (2x+1)$

$y' = \frac{2x+1}{2\left(x^2+x+13\right)^{1/2}}$

5. $D_x\left[\sqrt{1-4x^2}\right]$

$= D_x\left[\left(1-4x^2\right)^{1/2}\right]$

$= \frac{1}{2}\left(1-4x^2\right)^{-1/2} \bullet (-8x)$

$= \frac{-4x}{\left(1-4x^2\right)^{1/2}}$

7. $y = \sqrt{-x^4+41x^2-400}$

$\frac{dy}{dx} = \frac{1}{2}\left(-x^4+41x^2-400\right)^{-1/2}\left(-4x^3+82x\right)$

$$\frac{dy}{dx}=\frac{-2x^3+41x}{\sqrt{-x^4+41x^2-400}}$$

9. $y=\sqrt{-x^3-x^2+9x+9}$

$$\frac{dy}{dx}=\frac{1}{2}\left(-x^3-x^2+9x+9\right)^{-1/2}\left(-3x^2-2x+9\right)$$

$$\frac{dy}{dx}=\frac{-3x^2-2x+9}{2\sqrt{-x^3-x^2+9x+9}}$$

11. $y=\sqrt{x^2-5x-21}$

$$\frac{dy}{dx}=\frac{1}{2}\left(x^2-5x-21\right)^{-1/2}(2x-5)$$

$$\frac{dy}{dx}=\frac{2x-5}{2\sqrt{x^2-5x-21}}$$

13. $y=\sqrt{\frac{x}{x^2-4}}=\left(\frac{x}{x^2-4}\right)^{1/2}$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{x}{x^2-4}\right)^{-1/2}\bullet\left[\frac{\left(x^2-4\right)(1)-(x)(2x)}{\left(x^2-4\right)^2}\right]=\frac{1}{2}\left(\frac{x^2-4}{x}\right)^{1/2}\bullet\left[\frac{x^2-4-2x^2}{\left(x^2-4\right)^2}\right]$$

$$=\frac{1}{2}\bullet\frac{\left(x^2-4\right)^{1/2}}{x^{1/2}}\bullet\left[\frac{-x^2-4}{\left(x^2-4\right)^2}\right]=\frac{-x^2-4}{2x^{1/2}\left(x^2-4\right)^{3/2}}$$

15. $y=\sqrt{\dfrac{x^2-9}{x^2-4}}=\left(\dfrac{x^2-9}{x^2-4}\right)^{1/2}$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{x^2-9}{x^2-4}\right)^{-1/2}\bullet\left[\frac{\left(x^2-4\right)\left(2x\right)-\left(x^2-9\right)\left(2x\right)}{\left(x^2-4\right)^2}\right]=\frac{1}{2}\left(\frac{x^2-4}{x^2-9}\right)^{1/2}\bullet\left[\frac{2x^3-8x-2x^3+18x}{\left(x^2-4\right)^2}\right]$$

$$=\frac{1}{2}\bullet\frac{\left(x^2-4\right)^{1/2}}{\left(x^2-9\right)^{1/2}}\bullet\left[\frac{10x}{\left(x^2-4\right)^2}\right]=\frac{5x}{\left(x^2-9\right)^{1/2}\left(x^2-4\right)^{3/2}}$$

17. $y=-\sqrt{\dfrac{6x}{x^2+9}}=-\left(\dfrac{6x}{x^2+9}\right)^{1/2}$

$$\frac{dy}{dx}=-\frac{1}{2}\left(\frac{6x}{x^2+9}\right)^{-1/2}\bullet\left[\frac{\left(x^2+9\right)\left(6\right)-\left(6x\right)\left(2x\right)}{\left(x^2+9\right)^2}\right]=-\frac{1}{2}\left(\frac{x^2+9}{6x}\right)^{1/2}\bullet\left[\frac{6x^2+54-12x^2}{\left(x^2+9\right)^2}\right]$$

$$=-\frac{1}{2}\bullet\frac{\left(x^2+9\right)^{1/2}}{\left(6x\right)^{1/2}}\bullet\left[\frac{-2x^2+54}{\left(x^2+9\right)^2}\right]=\frac{x^2-27}{\left(6x\right)^{1/2}\left(x^2+9\right)^{3/2}}$$

19. $y=\sqrt{\dfrac{x^2+3x}{x^2+2x-3}}=\left(\dfrac{x^2+3x}{x^2+2x-3}\right)^{1/2}=\left(\dfrac{\left(x+3\right)x}{\left(x-1\right)\left(x+3\right)}\right)^{1/2}=\left(\dfrac{x}{x-1}\right)^{1/2}$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{x}{x-1}\right)^{-1/2}\bullet\left[\frac{\left(x-1\right)\left(1\right)-\left(x\right)\left(1\right)}{\left(x-1\right)^2}\right]=\frac{1}{2}\left(\frac{x-1}{x}\right)^{1/2}\bullet\left[\frac{-1}{\left(x-1\right)^2}\right]$$

$$=\frac{1}{2}\bullet\frac{\left(x-1\right)^{1/2}}{\left(x\right)^{1/2}}\bullet\left[\frac{-1}{\left(x^2+9\right)^2}\right]=\frac{-1}{2\left(x\right)^{1/2}\left(x-1\right)^{3/2}}$$

21. $y = \left(4x^2 - 5x + 1\right)^5$

$y' = 5\left(4x^2 - 5x + 1\right)^4 \bullet \left(8x - 5\right)$

23. $y = \sqrt[4]{1 - 2x}$

$y = \left(1 - 2x\right)^{1/4}$

$y' = \frac{1}{4}\left(1 - 2x\right)^{-3/4} \bullet \left(-2\right)$

$y' = \frac{-1}{2\left(1 - 2x\right)^{3/4}}$

25. $y = \sqrt[10]{16 - x^2}$

$y = \left(16 - x^2\right)^{1/10}$

$\frac{dy}{dx} = \frac{1}{10}\left(16 - x^2\right)^{-9/10} \bullet \left(-2x\right)$

$\frac{dy}{dx} = \frac{-x}{5\left(16 - x^2\right)^{9/10}}$

29. $y = \frac{1}{\sqrt{25 - x^2}}$

$y = \left(25 - x^2\right)^{-1/2}$

$y' = -\frac{1}{2}\left(25 - x^2\right)^{-3/2} \bullet \left(-2x\right)8.$

$y' = \frac{x}{\left(25 - x^2\right)^{3/2}}$

31. $\dfrac{d}{dx}\left[\sqrt{\dfrac{5x+6}{5x-4}}\right]$

$$= D_x\left[\left(\frac{5x+6}{5x-4}\right)^{1/2}\right]$$

$$= \frac{1}{2}\left(\frac{5x+6}{5x-4}\right)^{-1/2} \bullet \left[\frac{(5x-4)(5)-(5x+6)(5)}{(5x-4)^2}\right]$$

$$= \frac{1}{2}\left(\frac{5x-4}{5x+6}\right)^{1/2} \bullet \left[\frac{25x^2-20-25x^2+30}{(5x-4)^2}\right]$$

$$= \frac{1}{2} \bullet \frac{(5x-4)^{1/2}}{(5x+6)^{1/2}} \bullet \left[\frac{10}{(5x-4)^2}\right]$$

$$= \frac{5}{(5x+6)^{1/2}(5x-4)^{3/2}}$$

33. $D_x\left[\sqrt{\dfrac{2x^3-5x^2+2x}{x^2-4}}\right]$

$$= D_x\left[\left(\frac{2x^3-5x^2+2x}{x^2-4}\right)^{1/2}\right] = D_x\left[\frac{(x)(2x-1)(x-2)}{(x+2)(x-2)}\right]^{1/2} = D_x\left[\left(\frac{2x^2-x}{x+2}\right)^{1/2}\right]$$

$$= \frac{1}{2}\left(\frac{2x^2-x}{x+2}\right)^{-1/2} \bullet \left[\frac{(x+2)(4x-1)-(2x^2-x)(1)}{(x+2)^2}\right]$$

$$= \frac{1}{2}\left(\frac{2x^2-x}{x+2}\right)^{-1/2} \bullet \left[\frac{4x^2+7x-2-2x^2+x}{(x+2)^2}\right]$$

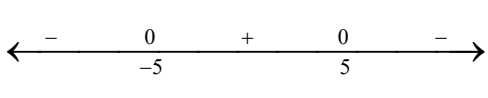
$$= \frac{1}{2}\left(\frac{2x^2-x}{x+2}\right)^{-1/2} \bullet \left[\frac{2x^2+8x-2}{(x+2)^2}\right]$$

$$= \left(\frac{x+2}{2x^2-x}\right)^{1/2} \bullet \left[\frac{x^2+4x-1}{(x+2)^2}\right]$$

$$= \frac{x^2+4x-1}{(2x^2-x)^{1/2}(x+2)^{3/2}}$$

<u>Section 8–2 Multiple Choice</u>

1. y^2



x

$a - x^2 \geq 0$

$a \geq x^2$

$\pm\sqrt{a} = x$

If $\pm\sqrt{a} = \pm 5$, then $a = 25$

(B) is the correct response

3. Find the slope of the line through the given two points using the slope formula:

$$m = \frac{3-1}{9-1}$$

$$m = \frac{1}{4}$$

Find the derivative of $y = \sqrt{x}$

$$y = x^{1/2}$$

$$y' = \frac{1}{2}x^{-1/2}$$

$$y' = \frac{1}{2\sqrt{x}}$$

Since y' represents the slope of the tangent line,

$$\frac{1}{4} = \frac{1}{2\sqrt{x}}$$

$$2\sqrt{x} = 4$$

$$\sqrt{x} = 2$$

$$x = 4$$

$$y\Big|_{x=4} = 2$$

Therefore, the point $(4,\ 2)$ is the desired point

(D) is the correct response

5. $f(x) = h(x^2 - 3)$

$$f'(x) = h'(x^2 - 3)\bullet(2x)$$

$$f'(2) = h'(2^2 - 3)\bullet(2\bullet 2)$$

$$f'(2) = h'(1)\bullet(4)$$

$$f'(2) = 4h'(1)$$

(B) is the correct response

Section 8–3: Extreme Points of Radical Functions

Free Response Solutions

1. $f(x)=\sqrt{6-x}$

$f(2)=2$, so the point of tangency is $(2,\ 2)$

$f'(x)=\frac{1}{2}(6-x)^{-1/2}\bullet(-1)$

$f'(x)=\frac{-1}{2(6-x)^{1/2}}$

$f'(2)=-\frac{1}{4}$, so the slope of the tangent line is $-\frac{1}{4}$, and the equation of the tangent line is $y-2=-\frac{1}{4}(x-2)$.

For a linear approximation of $\sqrt{4.1}$, the first task is to find x:

$\sqrt{4.1}=\sqrt{6-x}$

$4.1=6-x$

$x=1.9$

Then, plug $x=1.9$ into the tangent line equation:

$y-2=-\frac{1}{4}(1.9-2)$

$y=-\frac{1}{4}(-0.1)+2$

$y=2.025$

3. $y=(5-3x)^{3/2}$

$5-3x\geq 0$

$x\leq\frac{5}{3}$

y^2 $\quad + \quad 0 \quad -$

x $\quad \frac{5}{3}$

Domain: $x\in\left(-\infty,\ \frac{5}{3}\right]$

$$y' = \frac{3}{2}(5-3x)^{1/2} \bullet (-3)$$

$$y' = \frac{-9(5-3x)^{1/2}}{2}$$

(i) $-9(5-3x)^{1/2} = 0$

$$x = \frac{5}{3}$$

(ii) $2 = 0$

None

(iii) Restrictions: none

CV: $x = \frac{5}{3}$

5. $y = \sqrt{9x^3 - 4x^2 - 27x + 12}$

$$9x^3 - 4x^2 - 27x + 12 \geq 0$$

$$x^2(9x-4) - 3(9x-4) \geq 0$$

$$(9x-4)(x^2-3) \geq 0$$

$$x = \frac{4}{9},\ \pm\sqrt{3}$$

y^2	+	0	+	0	−	0	+
x		$-\sqrt{3}$		$\frac{4}{9}$		$\sqrt{3}$	

Domain: $x \in \left[-\sqrt{3},\ \frac{4}{9}\right] \cup \left(\sqrt{3},\ \infty\right)$

$$y' = \frac{1}{2}\left(9x^3 - 4x^2 - 27x + 12\right)^{-1/2} \bullet \left(27x^2 - 8x - 27\right)$$

$$y' = \frac{27x^2 - 8x - 27}{2\left(9x^3 - 4x^2 - 27x + 12\right)^{1/2}}$$

(i) $27x^2 - 8x - 27 = 0$

$x = 1.159,\ -0.863$ (Only -0.863 is in the domain of the function)

(ii) $\left(9x^3 - 4x^2 - 27x + 12\right)^{1/2} = 0$

$$x = \frac{4}{9},\ \pm\sqrt{3}$$

(iii) Restrictions: none

CVs: $x = \frac{4}{9},\ \pm\sqrt{3},\ -0.863$

7. $y = \sqrt{9 - x}$

$9 - x \geq 0$

$x \leq 9$

y^2 $\quad + \quad 0 \quad -$

$x \quad\quad 9$

Domain: $x \in (-\infty,\ 9]$

$$y' = \frac{1}{2}(9 - x)^{-1/2} \bullet (-1)$$

$$y' = \frac{-1}{2(9 - x)^{1/2}}$$

(i) $-1 = 0$

None

(ii) $2(9 - x)^{1/2} = 0$

$x = 9$

(iii) Restrictions: none

CVs: $x = 9$

9. $y = \sqrt{\frac{x^2 - 4}{x^2 - 1}}$

$$\frac{x^2 - 4}{x^2 - 1} \geq 0$$

Zeros: $x = \pm 2$; DNEs: $x = \pm 1$

y^2 $\quad + \quad 0 \quad - \quad DNE \quad + \quad DNE \quad - \quad 0 \quad +$

$x \quad\quad -2 \quad\quad -1 \quad\quad 1 \quad\quad 2$

Domain: $x \in (-\infty,\ -2] \cup (-1,\ 1) \cup [2,\ \infty)$

$$y' = \frac{1}{2}\left(\frac{x^2-4}{x^2-1}\right)^{-1/2} \bullet \left[\frac{\left(x^2-1\right)(2x)-\left(x^2-4\right)(2x)}{\left(x^2-1\right)^2}\right]$$

$$y' = \frac{\left(x^2-1\right)^{1/2}}{2\left(x^2-4\right)^{1/2}} \bullet \left[\frac{2x^3-2x-2x^3+8x}{\left(x^2-1\right)^2}\right]$$

$$y' = \frac{\left(x^2-1\right)^{1/2}}{2\left(x^2-4\right)^{1/2}} \bullet \frac{6x}{\left(x^2-1\right)^2}$$

$$y' = \frac{3x}{\left(x^2-4\right)^{1/2}\left(x^2-1\right)^{3/2}}$$

i. $3x = 0$

$x = 0$

ii. $2\left(x^2-4\right)^{1/2}\left(x^2-1\right)^{3/2} = 0$

$x = \pm 2,\ \pm 1$ (but ± 1 are not in the domain of this function)

iii. Restrictions: none

CVs: $x = 0,\ \pm 2$

11. $y = \sqrt{-x^4+41x^2-400} \rightarrow \dfrac{dy}{dx} = \dfrac{-2x^3+41x}{\sqrt{-x^4+41x^2-400}}$ (see 8-2 #7)

i. $-2x^3+41x = -x\left(2x^2+41\right) = 0 \rightarrow x = 0, \pm\sqrt{20.5}$, but $x = 0$ are not in the domain. Therefore, $x = \pm\sqrt{20.5}$.

ii. $-x^4+41x^2-400 = -\left(x^2-25\right)\left(x^2-16\right) = 0 \rightarrow x = \pm 4, \pm 5$

iii. Restrictions: none

$$\left(\pm 4,\ 0\right),\ \left(\pm 5,\ 0\right),\ \left(\pm\sqrt{20.5},\ 4.5\right)$$

13. $y = \sqrt{-x^3-x^2+9x+9} \rightarrow \dfrac{dy}{dx} = \dfrac{-3x^2-2x+9}{2\sqrt{-x^3-x^2+9x+9}}$ (see 8-2 #9)

i. $-3x^2-2x+9=0 \rightarrow x=\dfrac{2\pm\sqrt{4-4(-3)(9)}}{2(-3)}=1.431, -2.431$, but $x=-2.431$ are not in the domain. Therefore, $x=1.431$.

ii. $-x^3-x^2+9x+9=-x^2(x+1)+9(x+1)=(9-x^2)(x+1)=0 \rightarrow$ $x=\pm 3, -1$

iii. Restrictions: none

$$(\pm 3,\ 0),\ (-1,\ 0),\ (1.431,\ 4.111)$$

15. $y=\sqrt{x^2-5x-21} \rightarrow \dfrac{dy}{dx}=\dfrac{2x-5}{2\sqrt{x^2-5x-21}}$ (see 8-2 #11)

i. $2x-5=0 \rightarrow x=2.5$, but $x=2.5$ are not in the domain. Therefore, none.

ii. $x^2-5x-21=(x-7)(x+3)=0 \rightarrow x=7, -3$

iii. Restrictions: none

$$(7,\ 0),\ (-3,\ 0)$$

17. $y=\sqrt{\dfrac{-8x}{x^2+4}}=\left(\dfrac{-8x}{x^2+4}\right)^{1/2}$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{-8x}{x^2+4}\right)^{-1/2}\bullet\left[\frac{(x^2+4)(-8)-(-8x)(2x)}{(x^2+4)^2}\right]=-\frac{1}{2}\left(\frac{x^2+4}{-8x}\right)^{1/2}\bullet\left[\frac{-8x^2-32+16x^2}{(x^2+9)^2}\right]$$

$$=-\frac{1}{2}\bullet\frac{(x^2+4)^{1/2}}{(-8x)^{1/2}}\bullet\left[\frac{8x^2-32}{(x^2+4)^2}\right]=\frac{4(x^2-4)}{(-8x)^{1/2}(x^2+4)^{3/2}}$$

i. $x^2-4=0 \rightarrow x=\pm 2$, but $x=2$ are not in the domain. Therefore, $x=-2$.

ii. $(-8x)^{1/2}(x^2+4)^{3/2}=0 \rightarrow x=0$

iii. Restrictions: none

$$(0,0),\ \left(-2,\ \sqrt{2}\right)$$

19. $y=\sqrt{\dfrac{x^2-9}{x^2-4}} \rightarrow \dfrac{dy}{dx}=\dfrac{5x}{\left(x^2-9\right)^{1/2}\left(x^2-4\right)^{3/2}}$ (see 8-2 #15)

i. $5x=0 \rightarrow x=0$

ii. $\left(x^2-9\right)^{1/2}\left(x^2-4\right)^{3/2}=0 \rightarrow x=\pm 3, \pm 2$, but $x=\pm 2$ are not in the domain. Therefore, $x=\pm 3$.

iii. Restrictions: none

$$\left(0,\ \frac{3}{2}\right),\ (\pm 3,\ 0)$$

21. $y=-\sqrt{\dfrac{6x}{x^2+9}} \rightarrow \dfrac{dy}{dx}=\dfrac{x^2-27}{(6x)^{1/2}\left(x^2+9\right)^{3/2}}$ (see 8-2 #17)

i. $x^2-27=0 \rightarrow x=\pm 3\sqrt{3}$, but $x=-3\sqrt{3}$ is not in the domain. Therefore, $x=3\sqrt{3}$.

ii. $(6x)^{1/2}\left(x^2+9\right)^{3/2}=0 \rightarrow x=0$

iii. Restrictions: none

$$(0,\ 0),\ \left(3\sqrt{3},\ -\frac{3}{\sqrt{2}}\right)$$

23. $y=\sqrt{\dfrac{x^2+3x}{x^2+2x-3}} \rightarrow \dfrac{dy}{dx}=\dfrac{-1}{2(x)^{1/2}(x-1)^{3/2}}$ (see 8-2 #19)

i. $-1=0$: none.

ii. $(x)^{1/2}(x-1)^{3/2}=0 \rightarrow x=0,\,1$, bt neither are in the domain.

iii. Restrictions: none

No extreme points

25. $y=\sqrt[7]{x^3-2x}$

Domain: $x\in(-\infty,\ \infty)$ (the function is an odd root)

$$y'=\frac{1}{7}\left(x^3-2x\right)^{-6/7}\bullet\left(3x^2-2\right)$$

$$y'=\frac{3x^2-2}{7\left(x^3-2x\right)^{6/7}}$$

i. $3x^2-2=0$

$x=\pm\sqrt{\frac{2}{3}}$ or ± 0.816

ii. $7\left(x^3-2x\right)^{6/7}=0$

$x^3-2x=0$

$x\left(x^2-2\right)=0$

$x=0,\pm\sqrt{2}$ or ± 1.414

iii. Restrictions: None

y'	$-$	DNE	$+$	0	$-$	0	$-$	0	$+$	DNE	$+$
x		$-\sqrt{2}$		$-\sqrt{\frac{2}{3}}$		0		$\sqrt{\frac{2}{3}}$		$\sqrt{2}$	

$$y\left(\sqrt{\frac{2}{3}}\right)=-1.012$$

$$y\left(-\sqrt{\frac{2}{3}}\right)=1.012$$

Extreme Points: $(0.816,\ -1.012),\ (-0.816,\ 1.012)$

27. $f(x)=\sqrt[5]{\dfrac{2x-1}{x^2-4}}$

VAs at $x=\pm 2$; function has an odd root

Domain: $x=(-\infty\ -2)\cup(-2,\ 2)\cup(2,\ \infty)$

$$f'(x)=\frac{1}{5}\left(\frac{2x-1}{x^2-4}\right)^{-4/5}\bullet\left[\frac{(x^2-4)(2)-(2x-1)(2x)}{(x^2-4)^2}\right]$$

$$f'(x)=\frac{(x^2-4)^{4/5}}{5(2x-1)^{4/5}}\bullet\left[\frac{2x^2-8-4x^2+2x}{(x^2-4)^2}\right]$$

$$f'(x)=\frac{(x^2-4)^{4/5}}{5(2x-1)^{4/5}}\bullet\left[\frac{-2x^2+2x-8}{(x^2-4)^2}\right]$$

$$f'(x)=\frac{-2(x^2+x-4)}{5(2x-1)^{4/5}(x^2-4)^{6/5}}$$

i. $2(x^2-x+4)=0$

None

ii. $(2x-1)^{4/5}=0$ or $(x^2-4)^{6/5}=0$

$x=\dfrac{1}{2},\ \pm 2$, but $x=\pm 2$ are VAs

iii. Restrictions: None

y'	–	*DNE*	–	*DNE*	–	*DNE*	–	
x		-2		$\frac{1}{2}$		2		

There are no Extreme Points.

29. $y=\sqrt{9-4x^2}$

$9-4x^2\geq 0$

$(3+2x)(3-2x)\geq 0$

$x=\ \pm\dfrac{3}{2}$

y^2 $\quad - \quad 0 \quad + \quad 0 \quad -$

x $\quad -\frac{3}{2} \quad \frac{3}{2}$

Domain: $x \in \left(-\frac{3}{2}, \frac{3}{2}\right]$

$$y' = \frac{1}{2}\left(9-4x^2\right)^{-1/2} \bullet (-8x)$$

$$y' = \frac{-4x}{\left(9-4x^2\right)^{1/2}}$$

(i) $-4x = 0$

$x = 0$

(ii) $\left(9-4x^2\right)^{1/2} = 0$

$x = \pm\frac{3}{2}$

(iii) Restrictions

None

CVs: $x = 0, \pm\frac{3}{2}$

y' $\quad DNE \quad + \quad 0 \quad - \quad DNE$

x $\quad -\frac{3}{2} \quad 0 \quad \frac{3}{2}$

$x = 0$ is at a max

$x = -\frac{3}{2}$ and $x = \frac{3}{2}$ are both at mins

31. $F(x) = \sqrt{\frac{3x}{x^2+9}}$

$$F'(x) = \frac{1}{2}\left(\frac{3x}{x^2+9}\right)^{-1/2}\left[\frac{\left(x^2+9\right)(3)-3x(2x)}{\left(x^2+9\right)^2}\right] = \frac{1}{2}\frac{\left(x^2+9\right)^{1/2}}{(3x)^{1/2}}\left[\frac{-3x^2+27}{\left(x^2+9\right)^2}\right]$$

$$= \frac{-3x^2+27}{2(3x)^{1/2}\left(x^2+9\right)^{3/2}}$$

i. $\frac{dy}{dx} = 0 \rightarrow -3x^2+27 = 0 \rightarrow x = \cancel{-3}, 3$

ii. $\frac{dy}{dx} = dne \rightarrow 2(3x)^{1/2}(x^2+9)^{3/2} = 0 \rightarrow x = 0$

iii. No endpoints given

$$\left(3,\ \frac{1}{\sqrt{2}}\right),\ (0,\ 0)$$

33. $x(t) = \sqrt{5+t^2}$

$v(t) = \frac{1}{2}(5+t^2)^{-1/2} \bullet (2t)$

$v(t) = \frac{t}{(5+t^2)^{1/2}}$

$v(2) = \frac{2}{\sqrt{9}} = \frac{2}{3}$

35. $x(t) = \sqrt{t^2+4}$

$y(t) = \sqrt{10t - t^2}$

(a)

$v_x(t) = \frac{1}{2}(t^2+4)^{-1/2} \bullet (2t)$

$v_x(t) = t(t^2+4)^{-1/2}$

$v_x(2) = 0.707$

$v_y(t) = \frac{1}{2}(10t - t^2)^{-1/2} \bullet (10 - 2t)$

$v_y(t) = (5-t)(10t - t^2)^{-1/2}$

$v_y(2) = \frac{3}{4}$

$\therefore \left(0.707,\ \frac{3}{4}\right)$

(b)

$$a_x(t) = t \bullet \left[-\frac{1}{2}\left(t^2+4\right)^{-3/2} \bullet (2t) \right] + \left[\left(t^2+4\right)^{-1/2} \bullet (1) \right]$$

$$a_x(t) = -t^2\left(t^2+4\right)^{-3/2} + \left(t^2+4\right)^{-1/2}$$

$$a_x(2) = 0.177$$

$$a_y(t) = \left[\left(10t - t^2\right)^{-1/2} \bullet (-1) \right] + \left[(5-t) \bullet \left(-\frac{1}{2}\left(10t - t^2\right)^{-3/2} \bullet (10-2t) \right) \right]$$

$$a_y(t) = -\left(10t - t^2\right)^{-1/2} - \left[(5-t)^2 \left(10t - t^2\right)^{-3/2} \right]$$

$$a_y(2) = -0.391$$

$$\therefore (0.177,\ -0.391)$$

<u>Section 8–3 Multiple Choice</u>

1. The sign pattern of $\frac{dy}{dx}$ changes for – to + at $x = 0$

   (C) is the correct response

3. $y=\sqrt{36-x^2}$

$y'=\frac{1}{2}\left(36-x^2\right)^{-1/2}\bullet\left(-2x\right)$

$y'=\frac{-x}{\left(36-x^2\right)^{1/2}}$

(i) $-x=0$

$x=0$

(ii) $\left(36-x^2\right)^{1/2}=0$

$x=\pm 6$ (but these are not in the stated domain)

(iii) Restrictions

$x=\pm 2$

Extreme Values:

$y\Big|_{x=0}=6$

$y\Big|_{x=2}=4\sqrt{2}$

$y\Big|_{x=-2}=4\sqrt{2}$

The largest of these is $4\sqrt{2}$

(B) is the correct response

5. $h'(x)=f'\left(g(x)\right)\bullet g'(x)$

$h'(2)=f'\left(g(2)\right)\bullet g'(2)$

$h'(2)=f'(-1)\bullet 2$

$h'(2)=3\bullet 2$

$h'(2)=6\neq$

(D) is the correct response

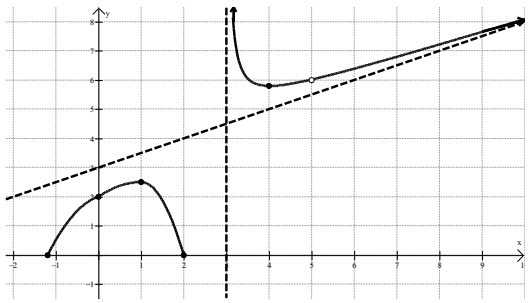
7. Sign Patterns II and III apply

(D) is the correct response

Section 8–4: General Radical Curve Sketching

Section 8–4 Solutions

1.



3. $y=\sqrt{x^2-4x-5}$

$x^2-4x-5\geq 0$

$(x-5)(x+1)\geq 0$

$x=5,\ -1$

y^2 $\quad + \quad 0 \quad - \quad 0 \quad +$

x $\quad -1 \quad 5$

$y'=\frac{1}{2}\left(x^2-4x-5\right)^{-1/2}\bullet(2x-4)$

$y'=\dfrac{x-2}{\left(x^2-4x-5\right)^{1/2}}$

(i) $x-2=0$

$x=2$, but this is not in the domain

(ii) $\left(x^2-4x-5\right)^{1/2}=0$

$x=-1,\ 5$

(iii) Restrictions:

None

y' $\quad - \quad DNE \quad - \quad 0 \quad + \quad DNE \quad +$

x $\quad -1 \quad 2 \quad 5$

Left EB:

$\lim\limits_{x\to-\infty}\sqrt{x^2-4x-5}$

$= \infty$ (up)

Right EB:

$$\lim_{x \to \infty} \sqrt{x^2 - 4x - 5}$$

$= \infty$ (up)

Domain: $x \in (-\infty,\ -1] \cup [5,\ \infty)$

Range: $y \in [0,\ \infty)$

Zeros: $(-1,\ 0),\ (5,\ 0)$

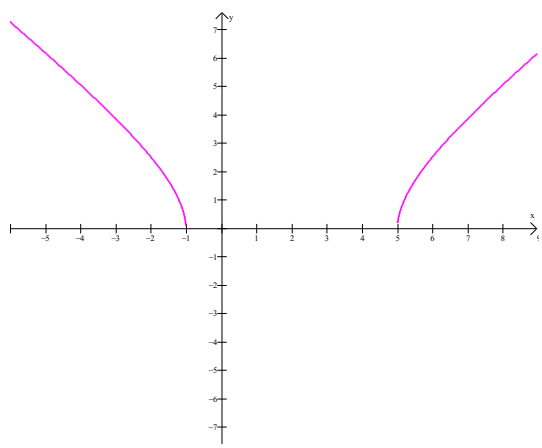
y-int: None

EB: Both ends up

VA: None

POE: None

Extreme Points: $(-1,\ 0),\ (5,\ 0)$

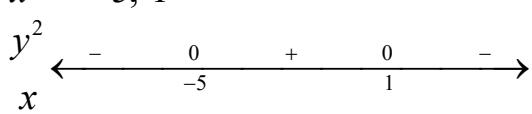


5. $y = \sqrt{5 - 4x - x^2}$

$5 - 4x - x^2 = 0$

$(5 + x)(1 - x) = 0$

$x = \ -5,\ 1$



$$y' = \frac{1}{2}\left(5 - 4x - x^2\right)^{-1/2} \bullet \left(-4 - 2x\right)$$

$$y' = \frac{-2 - x}{\left(5 - 4x - x^2\right)^{1/2}}$$

(i) $-2 - x = 0$

$x = -2$

(ii) $5 - 4x - x^2 = 0$

$x = -5,\ 1$

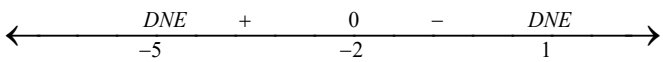
(iii) Restrictions:

None

$$y\Big|_{x=-2} = 3$$

y'

x



Domain: $x \in \left[-5,\ 1\right]$

Range: $y \in \left[0,\ 3\right]$

Zeros: $\left(-5,\ 0\right),\ \left(1,\ 0\right)$

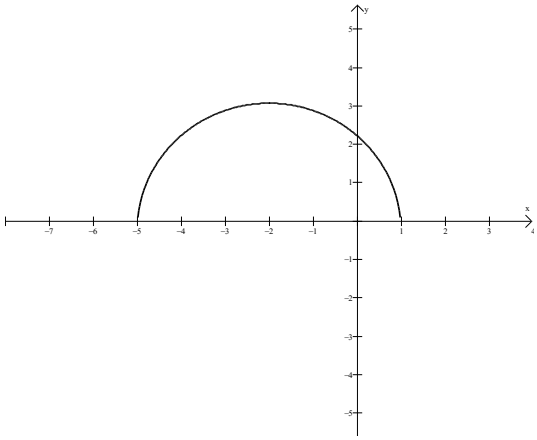
y-int: $\left(0,\ \sqrt{5}\right)$

EB: None (because neither $-\infty$ nor ∞ is in the domain)

VA: None

POE: None

Extreme Points: $\left(-2,\ 3\right),\ \left(-5,\ 0\right),\ \left(1,\ 0\right)$



7. $y=\sqrt{x^3+3x^2-6x-8}$ on $x\in[-4,\ 4]$

$x^3+3x^2-6x-8\geq 0$

$(x-2)(x+4)(x+1)\geq 0$

$x=2,\ -4,\ -1$

y^2	$END(0)$	$+$	0	$-$	0	$+$	END
x	-4		-1		2		4

$$y'=\frac{1}{2}\left(x^3+3x^2-6x-8\right)^{-1/2}\bullet\left(3x^2+6x-6\right)$$

$$y'=\frac{3x^2+6x-6}{2\left(x^3+3x^2-6x-8\right)^{1/2}}$$

(i) $3x^2+6x-6=0$

$x=0.732,\ -2.732$ (0.732 is not in the domain)

(ii) $2\left(x^3+3x^2-6x-8\right)^{1/2}=0$

$x=2,\ -4,\ -1$

(iii) Restrictions:

$x=-4,\ 4$

$$y\Big|_{x=-2.732}=3.224$$

$$y\Big|_{x=4}=8.944$$

y'	0	$+$	0	$-$	DNE		DNE	$+$	END
x	-4		-2.732		-1		2		4

Domain: $x\in[-4,\ -1]\cup[2,\ 4]$

Range: $y\in[0,\ 8.994]$

Zeros: $(-4,\ 0),\ (-1,\ 0),\ (2,\ 0)$

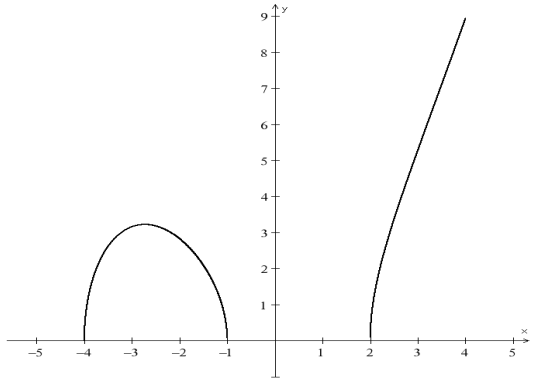
y-int: None

EB: None (because neither $-\infty$ nor ∞ is in the domain)

VA: None

POE: None

Extreme Points: $(-4,\ 0),(-2.732,\ 3.224),\ (-1,\ 0),\ (2,\ 0),(4,\ 8.994)$



9. $y = -\sqrt{\dfrac{x^2-1}{x^2-4}}$

$\dfrac{x^2-1}{x^2-4} \geq 0$

Zeros: $(\pm 1,\ 0)$; VAs: $x = \pm 2$

y^2	+	*DNE*	−	0	+	0	−	*DNE*	+
x		-2		-1		1		2	

$$y = \left(\frac{x^2-1}{x^2-4}\right)^{1/2}$$

$$y' = -\frac{1}{2}\left(\frac{x^2-1}{x^2-4}\right)^{-1/2} \bullet \left[\frac{(x^2-4)(2x)-(x^2-1)(2x)}{(x^2-4)^2}\right]$$

$$y' = \frac{-(x^2-4)^{1/2}}{2(x^2-1)^{1/2}} \bullet \frac{-6x}{(x^2-4)^2}$$

$$y' = \frac{3x}{(x^2-1)^{1/2}(x^2-4)^{3/2}}$$

(i) $3x = 0$

$x = 0$

(ii) $(x^2-1)^{1/2}(x^2-4)^{3/2} = 0$

$x = \pm 1,\ \pm 2$ (± 2 are not in the domain of the function)

(iii) Restrictions:

None

y'	$-$	DNE		DNE	$-$	0	$+$	DNE		DNE	$+$
x		-2		-1		0		1		2	

Domain: $x \in (-\infty,\ -2) \cup [-1,\ 1] \cup (2,\ \infty)$

Range: $y \in (-\infty,\ -1) \cup \left[-\frac{1}{2},\ 0\right]$

Zeros: $(1,\ 0),\ (-1,\ 0)$

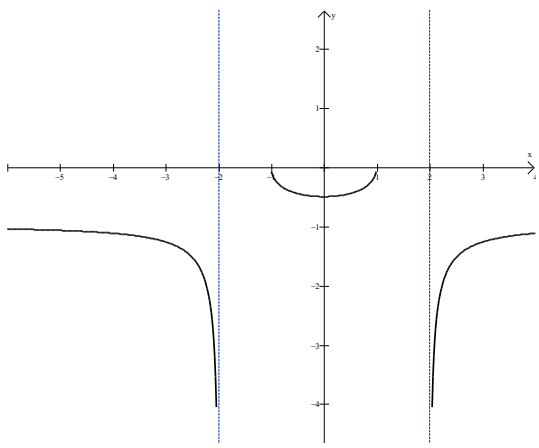
y-int: $\left(0,\ -\frac{1}{2}\right)$

VA: $x = -2,\ x = 2$

POE: None

EB: HA at $y = -1$ (both left and right)

Extreme Points: $\left(0,\ -\frac{1}{2}\right),\ (1,\ 0),\ (-1,\ 0)$



11. $y = -\sqrt{x^4 - 3x^2 - 10}$ on $x \in [-5,\ 3]$

$x^4 - 3x^2 - 10 \geq 0$

$(x^2 - 5)(x^2 + 2) \geq 0$

$x = \pm\sqrt{5}\ \ (x = \ \pm 2.236)$

y^2	END	$+$	0	$-$	0	$+$	END
x	-5		-2.236		2.236		3

$y = -\left(x^4 - 3x^2 - 10\right)^{1/2}$

$$y' = -\frac{1}{2}\left(x^4 - 3x^2 - 10\right)^{-1/2} \bullet \left(4x^3 - 6x\right) = \frac{-2x^3 + 3x}{\left(x^4 - 3x^2 - 10\right)^{1/2}} = \frac{-x\left(2x^2 + 3\right)}{\left(x^4 - 3x^2 - 10\right)^{1/2}}$$

i) $-x\left(2x^2 + 3\right) = 0$

$x = 0,\ \pm\sqrt{\frac{3}{2}}$ (none of these are in the domain of the function)

ii) $\left(x^4 - 3x^2 - 10\right)^{1/2} = 0$

$x = \pm\sqrt{5}$

iii) Restrictions:

$x = -5,\ 3$

$y(-5) = -23.238 \quad y(3) = -6.633$

y'	END	$+$	DNE		DNE	$-$	END
x	-5		$-\sqrt{5}$		$\sqrt{5}$		3

Domain: $x \in \left[-5,\ -\sqrt{5}\right] \cup \left[\sqrt{5},\ 3\right]$

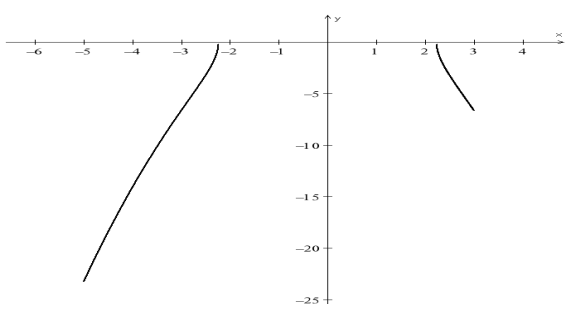
Range: $y \in \left[-23.238,\ 0\right]$

Zeros: $\left(\pm\sqrt{5},\ 0\right)$

y-int: None VA: None POE: None

EB: None (because neither $-\infty$ nor ∞ is in the domain)

Extreme Points: $\left(\pm\sqrt{5},\ 0\right),\ (-5,\ -23.238),\ (3,\ -6.633)$



13. $F(x) = \sqrt{\frac{3x}{x^2 + 9}}$ on $x \in \left[0,\ 4\right]$

Domain: Given as $x \in \left[0,\ 4\right]$ Zeros: $(0,\ 0)$ y-int: $(0,\ 0)$

VA: $x^2 + 9 \neq 0$ None POE: None

EB: None because of the domain

Extreme Points:

$$F'(x)=\frac{1}{2}\left(\frac{3x}{x^2+9}\right)^{-1/2}\left[\frac{(x^2+9)(3)-3x(2x)}{(x^2+9)^2}\right]=\frac{1}{2}\frac{(x^2+9)^{1/2}}{(3x)^{1/2}}\left[\frac{-3x^2+27}{(x^2+9)^2}\right]$$

$$=\frac{-3x^2+27}{2(3x)^{1/2}(x^2+9)^{3/2}}$$

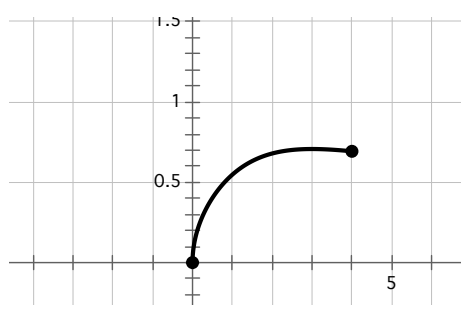
$i)\quad \frac{dy}{dx}=0\rightarrow -3x^2+27=0\rightarrow x=\cancel{-3},\ 3$

$ii)\quad \frac{dy}{dx}=dne\rightarrow 2(3x)^{1/2}(x^2+9)^{3/2}=0\rightarrow x=0$

$iii)\quad x=0,\ 4$

$$(0,\ 0),\left(3,\ \frac{1}{\sqrt{2}}\right),\left(4,\ \frac{2\sqrt{3}}{5}\right)$$

Range: $y\in\left[0,\ \frac{1}{\sqrt{2}}\right]$



15. $y=\sqrt{\frac{2x^3+x^2-32x-16}{2x^2-5x-3}}=\sqrt{\frac{x^2(2x+1)-16(2x+1)}{(2x+1)(x-3)}}\approx\sqrt{\frac{x^2-16}{x-3}}$

y^2	END	$+$	POE	$+$	VA	$-$	END
x	-4		$-\frac{1}{2}$		3		4

Domain: $x \in \left[-4, -\tfrac{1}{2}\right) \cup \left(-\tfrac{1}{2}, 3\right)$

Zeros: $(-4,\ 0)$ $\qquad$ y-int: $\left(0,\ \tfrac{4}{\sqrt{3}}\right)$

VA: $x = 3$ $\qquad$ POE: $\left(-\dfrac{1}{2},\ 1.5\right)$ $\qquad$ EB: None

Extreme Points:

$$\frac{d}{dx}\sqrt{\frac{x^2-16}{x-3}} = \frac{1}{2}\left(\frac{x^2-16}{x-3}\right)^{-1/2}\left[\frac{(x-3)(2x)-(x^2-16)(1)}{(x-3)^2}\right] = \frac{1}{2}\frac{(x-3)^{1/2}}{(x^2-16)^{1/2}}\left[\frac{(x^2-6x+16)}{(x-3)^2}\right]$$

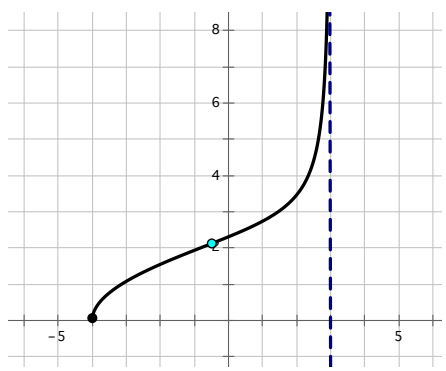
$i)\quad \dfrac{dy}{dx} = 0 \rightarrow x^2-6x+16=0 \rightarrow x = \dfrac{6 \pm \sqrt{6^2-4(1)(16)}}{2(1)} = \textit{no solution}$

$ii)\quad \dfrac{dy}{dx} = dne \rightarrow 2(x^2-16)^{1/2\,1/2}(x-3)^{3/2} = 0 \rightarrow x = \pm 4, 3$

$iii)\quad x = \pm 4$

$(-4,\ 0)$

Range: $y \in [0,\ 2.121) \cup (2.121, \infty)$



17. $y=\sqrt{-x^2+5x-6}$

$-x^2+5x-6\geq 0 \rightarrow -\left(x^2-5x+6\right)\geq 0 \rightarrow -(x-2)(x-3)\geq 0$

y^2	+	0	+	0	−
x		2		3	

Domain: $x\in\left[2,\ 3\right]$ Zeros: $\left(2,\ 0\right),\ \left(3,\ 0\right)$ y-int: None

VA: None POE: None EB: None

Extreme Points:

$$\frac{dy}{dx}=\frac{1}{2}\left(-x^2+5x-6\right)^{-1/2}\left[-2x+5\right]=\frac{\left(-2x+5\right)}{2\left(-x^2+5x-6\right)^{1/2}}$$

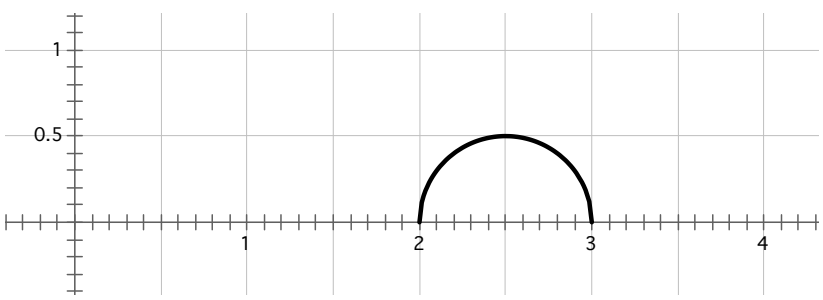
i) $\frac{dy}{dx}=0\rightarrow -2x+5=0\rightarrow x=\frac{5}{2}$, but 2.482 is not in the domain

ii) $\frac{dy}{dx}=dne\rightarrow -x^2+5x-6=0\rightarrow x=2,3$

iii) No endpoints given

$$\left(\frac{5}{2},\ \frac{1}{2}\right),\ \left(2,\ 0\right),\ \left(3,\ 0\right)$$

Range: $y\in\left[0,\ .5\right]$



19. $y=\sqrt{4x^4-37x^2+9}$ on $x\in\left[-2,\ 2\right]$

Domain: $4x^4-37x^2+9\geq 0\rightarrow\left(2x^2-1\right)\left(x^2-9\right)\geq 0$

y^2	+	0	−	0	+	0	−	0	+
x		-3		$-\frac{1}{2}$		$\frac{1}{2}$		3	

$$\left[-4,\ -3\right]\cup\left[-\frac{1}{2},\ -\frac{1}{2}\right]\cup\left[3,\ 4\right]$$

Zeros: $\left(\pm\frac{1}{2},\ 0\right),\left(\pm 3, 0\right)$ y-int: $\left(0,\ 3\right)$

VA: None POE: None EB: None

Extreme Points: $\frac{dy}{dx}=\frac{1}{2}\left(4x^4-37x^2+9\right)^{-1/2}\left[16x^3-74x\right]=\frac{8x^3-37x}{\left(4x^4-37x^2+9\right)^{1/2}}$

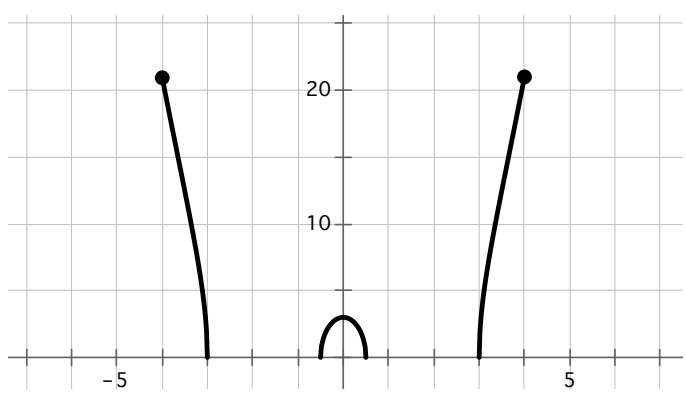
i) $\frac{dy}{dx}=0\rightarrow 8x^3-37x=0\rightarrow x=0,\pm 2.151$, but ± 2.151 are not in the domain

ii) $\frac{dy}{dx}=dne\rightarrow 4x^4-37x^2+9=0\rightarrow x=\pm\frac{1}{2},\pm 3$

iii) No endpoints given

$$\left(\pm\frac{1}{2},\ 0\right),\left(\pm 3, 0\right),\left(\pm 4,\ 21\right),\left(0,\ 3\right)$$

Range: $y\in\left[0,\ 21\right]$



Section 8–4 Multiple Choice

1. $y=\sqrt{81-x^2}$

$y'=\frac{1}{2}\left(81-x^2\right)^{-1/2}\bullet\left(-2x\right)$

$y'=\frac{-x}{\left(81-x^2\right)^{1/2}}$

(i) $-x=0$

$x=0$

(ii) $\left(81-x^2\right)^{1/2}=0$

$x=\ \pm 9$, but these are not in the stated domain

(iii) Restrictions

$x=\ \pm 3$

$y\Big|_{x=0}=9$

$y\Big|_{x=3}=8.485$

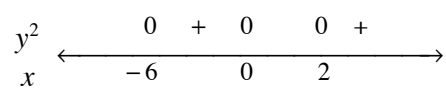
$y\Big|_{x=-3}=8.485$

The largest of these is $y=9$

(C) is the correct response

3. $y=\sqrt{12x-4x^2-x^3}$

$12x-4x^2-x^3=-x\left(x+6\right)\left(x-2\right)$



No end behavior on the right because of the domain.
The radical is positive, so the left end behavior is up.

(C) is the correct response

5. D) I and III only

I is correct because the sign pattern is for y, and y is positive in the correct intervals and non-existent in the others

II is incorrect because y^2 should have negatives where the curve is non-existent.

III is correct $\frac{dy}{dx} < 0$ because the curve is decreasing where there is a curve.

Section 8–5: Implicit Differentiation

Free Response Solutions

1. $$2x^2 + 2y^2 - 10x + 2y - 5 = 0$$
$$4x + 4y\left(\frac{dy}{dx}\right) - 10 + 2\left(\frac{dy}{dx}\right) = 0$$
$$4y\left(\frac{dy}{dx}\right) + 2\left(\frac{dy}{dx}\right) = -4x + 10$$
$$\frac{dy}{dx}(4y + 2) = -4x + 10$$
$$\frac{dy}{dx} = \frac{-4x + 10}{4y + 2}$$
$$\frac{dy}{dx} = \frac{-2x + 5}{2y + 1}$$

3. $$9x^2 + 4y^2 + 36x - 8y + 4 = 0$$
$$18x + 8y\left(\frac{dy}{dx}\right) + 36 - 8\left(\frac{dy}{dx}\right) = 0$$
$$18x + 36 = \frac{dy}{dx}(8 - 8y)$$
$$\frac{18(x + 2)}{8(1 - y)} = \frac{dy}{dx}$$
$$\frac{9(x + 2)}{4(1 - y)} = \frac{dy}{dx}$$

5. $$y^2 = \frac{x - 1}{x + 1}$$

$$2y\left(\frac{dy}{dx}\right)=\frac{(x+1)(1)-(x-1)(1)}{(x+1)^2}$$

$$2y\left(\frac{dy}{dx}\right)=\frac{2}{(x+1)^2}$$

$$\frac{dy}{dx}=\frac{1}{y(x+1)^2}$$

7. $$\frac{d}{dx}\left[x^2+y^2=\frac{x}{y}\right]\Rightarrow 2x+2y\frac{dy}{dx}=\frac{y(1)-x\left(\frac{dy}{dx}\right)}{y^2}\Rightarrow 2xy^2+2y^3\frac{dy}{dx}=y-x\frac{dy}{dx}$$

$$2y^3\frac{dy}{dx}+x\frac{dy}{dx}=y-2xy^2$$

$$\frac{dy}{dx}=\frac{y-2xy^2}{2y^3+x}$$

9. $$\frac{d}{dx}\left[y^2=\frac{x^2-1}{x+2}\right]\Rightarrow 2y\frac{dy}{dx}=\frac{(x+2)(2x)-(x^2-1)(1)}{(x+2)^2}=\frac{x^2+4x+1}{(x+2)^2}$$

$$\frac{dy}{dx}=\frac{x^2+4x+1}{2y(x+2)^2}$$

11. $x^2 + y^2 + 6x - 14y + 58 = 0$

$$2x + 2y\left(\frac{dy}{dx}\right) + 6 - 14\left(\frac{dy}{dx}\right) = 0$$

$$2x + 6 = \frac{dy}{dx}(14 - 2y)$$

$$\frac{2x+6}{14-2y} = \frac{dy}{dx}$$

$$\frac{x+3}{7-y} = \frac{dy}{dx}$$

At the point $(0,\ 8)$:

$$\frac{dy}{dx} = -3$$

$$y - 8 = -3(x - 0)$$

13. $x^2 + 25y^2 + 6x - 100y + 9 = 0$

$$2x + 50y\left(\frac{dy}{dx}\right) + 6 - 100\left(\frac{dy}{dx}\right) = 0$$

$$2x + 6 = \frac{dy}{dx}(100 - 50y)$$

$$\frac{2x+6}{100-50y} = \frac{dy}{dx}$$

$$\frac{x+3}{50-25y} = \frac{dy}{dx}$$

At the point $(3,\ 3.6)$:

$$\frac{dy}{dx} = \frac{6}{-40} = -\frac{3}{20}$$

$$y - 3.6 = -\frac{3}{20}(x - 3)$$

15. $9x^2 + 4y^2 + 36x - 8y + 4 = 0$

From Problem FR3:

$$\frac{dy}{dx} = \frac{9(x+2)}{4(1-y)}$$

At the point $(0,\ -2)$:

$$\frac{dy}{dx} = \frac{9(2)}{4(3)} = \frac{3}{2}$$

$$y + 2 = \frac{3}{2}(x - 0)$$

<u>Section 8–5 Multiple Choice</u>

1. $y^2 - 3x = 7$

$$2y\left(\frac{dy}{dx}\right) - 3 = 0$$

$$\frac{dy}{dx} = \frac{3}{2y}$$

(D) is the correct response

3. $2x + 2y\left(\frac{dy}{dx}\right) = 0$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\frac{dy}{dx} = \frac{-5}{-12} = \frac{5}{12}$$

$$y + 12 = \frac{5}{12}(x - 5)$$

$$12y + 144 = 5x - 25$$

$$5x - 12y = 169$$

(C) is the correct response

5. $x^3 - y^2 + x^2 - 1 = 0$

$$\frac{d}{dx}\left[x^3 - y^2 + x^2 - 1 = 0\right]$$

$$3x^2 - 2y\frac{dy}{dx} + 2x = 0$$

$$\frac{dy}{dx} = \frac{3x^2 + 2x}{2y}$$

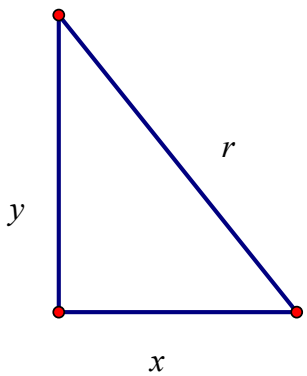
Horizontal tangent line means $\frac{dy}{dx} = 0$, so $3x^2 + 2x = 0 \rightarrow x(3x+2) = 0$

D) is the correct response

Section 8–6: Related Rates

Free Response Solutions

1a.



b) $x, y,\ r, \dfrac{dx}{dt}, \dfrac{dy}{dt}, and\ \dfrac{dr}{dt}$

c) $x = 1,\ y = 1.4,\ \dfrac{dx}{dt} = 5,\ \dfrac{dy}{dt} = 12,$

$1^2 + 1.4^2 = r^2 \rightarrow r = 1.720.$

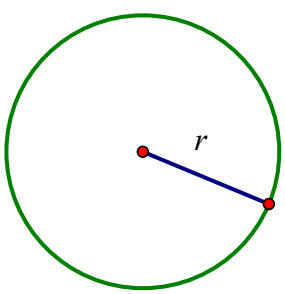
Find $\dfrac{dr}{dt}$

d) $x^2 + y^2 = r^2$

e) $\dfrac{d}{dt}\left[x^2 + y^2 = r^2\right] \rightarrow 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 2r\dfrac{dr}{dt} \rightarrow x\dfrac{dx}{dt} + y\dfrac{dy}{dt} = r\dfrac{dr}{dt}$

$(1)(5) + (1.4)(12) = (1.720)\dfrac{dr}{dt} \rightarrow \dfrac{dr}{dt} = \dfrac{21.8}{1.720} = 12.674 mph$

3a.



b) $A,\ r,\ \dfrac{dA}{dt}, and\ \dfrac{dr}{dt}$

c) $\dfrac{dr}{dt} = \dfrac{1}{2}\ \text{in}/\text{min},\ d = 1\ \text{in}(\ r = \dfrac{1}{2}\ \text{in})$

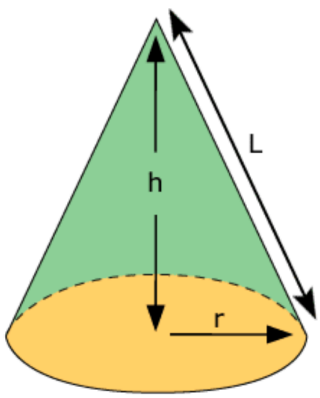
Find $\dfrac{dA}{dt}$

d) $A = \pi r^2$

e) $A = \pi r^2 \rightarrow \frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right) \rightarrow$

$$\frac{dA}{dt} = 2\pi\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{\pi}{2} \ \text{in}^2/\text{min}$$

5a. b) $Volume, diameter, height, radius, \frac{dV}{dt}, and \ \frac{dh}{dt}$



c) $\frac{dV}{dt} = 30\pi \ ft^3/\text{min}, \ h = 5$

$h = 5 \rightarrow r = 2.5$

Find $\frac{dh}{dt}$

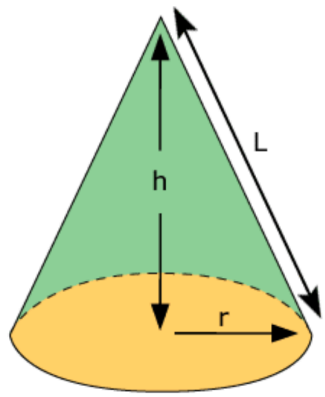
d) $\frac{1}{2}h = r; \ V = \frac{\pi}{3}r^2 h = \frac{\pi}{12}h^3;$

$$V = \frac{\pi}{3}r^2 h = \frac{\pi}{12}h^3$$

e) $\frac{d}{dt}\left[V = \frac{\pi}{12}h^3\right] \rightarrow \frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$$\frac{dV}{dt} = 30 = \frac{\pi}{4}10^2\frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{6}{5\pi}$$

7a)



b) $h, d, r, V, \frac{dh}{dt}, \frac{dr}{dt}, \frac{dV}{dt}$

c) $\frac{dV}{dt} = -5000 \ \text{cm}^3/\text{min}$; $h = 8$; $d = 4$

$d = 4 \rightarrow r = 2$

Find $\frac{dh}{dt}$ and $\frac{dr}{dt}$

d) $V = \frac{\pi}{3} r^2 h, \ d = 2r$

e) $\frac{r}{h} = \frac{2}{8} = \frac{1}{4} \rightarrow r = \frac{1}{4} h \rightarrow V = \frac{\pi}{3}\left(\frac{1}{4} h\right)^2 h = \frac{\pi}{48} h^3$

$$\frac{d}{dt}\left[V == \frac{\pi}{48} h^3\right] \rightarrow \frac{dV}{dt} = \frac{\pi}{16} h^2 \frac{dh}{dt}$$

$$-5000 = \frac{\pi}{16}\left(8^2\right)\frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{1250}{\pi} \ \text{m}^3/\text{min}$$

f) $\frac{r}{h} = \frac{1}{4} \rightarrow 4r = h \rightarrow V = \frac{\pi}{3} r^2 \left(4r\right) = \frac{4\pi}{3} r^3$

$$\frac{d}{dt}\left[V == \frac{4\pi}{3} r^3\right] \rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-5000 = 4\pi\left(2^2\right)\frac{dr}{dt}$$

$$\frac{dr}{dt} = -\frac{625}{2\pi}\ \text{m}^3/\text{min}$$

9. $\frac{dV}{dt} = 6\ \text{ft}^3/\text{min};\quad r = 10\ \text{ft};\ \frac{dr}{dt} = ?$

$$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$$

$$6 = 4\pi\left(10\right)^2\left(\frac{dr}{dt}\right)$$

$$\frac{6}{400\pi} = \frac{dr}{dt}$$

$$\frac{3}{200\pi}\ \text{ft}/\text{min} = \frac{dr}{dt}$$

11. $\frac{dx}{dt} = 5;\ y = 7;\ x^2 + 7^2 = 25^2 \rightarrow x = 24$

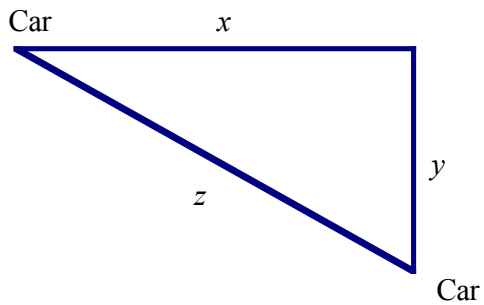
$$\frac{d}{dt}\left(x^2 + y^2 = 25\right)$$

$$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 0$$

$$2\left(24\right)\left(5\right) + 2\left(7\right)\left(\frac{dy}{dt}\right) = 0$$

$$\frac{dy}{dt} = -\frac{120}{7}\ ft/\text{sec}$$

13. $x^2 + y^2 = z^2;\quad \frac{dy}{dt} = 60\ mi/h;\quad \frac{dx}{dt} = 25\ mi/h$



$$\Rightarrow 2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$$

$$\Rightarrow 2\cdot 50\cdot 25+2\cdot 120\cdot 60=2\cdot 130\cdot\frac{dz}{dt}$$

$$\frac{dz}{dt}=65\,{}^{mi}\!/_{h}$$

15a. Person A: $Dist\ a=220-10\ (10)=120\,ft$

Person B: $Dist\ b=10\ (5)=50\,ft$

15b. $Dist\ c=\sqrt{50^2+120^2}=130\,ft$

15c. $\frac{d}{dt}\left[a^2+b^2=c^2\right]\rightarrow 2a\frac{da}{dt}+2b\frac{db}{dt}=2c\frac{dc}{dt}$

$$2(120)(-10)+2(50)(5)=2(130)\frac{dc}{dt}\rightarrow\frac{dc}{dt}=-7.308\,{}^{ft}\!/_{\sec}$$

15d. $\tan\theta=\frac{b}{a}\rightarrow\frac{d}{dt}\left[\tan\theta=\frac{b}{a}\right]\rightarrow\sec^2\theta\frac{d\theta}{dt}=\frac{a\frac{db}{dt}-b\frac{da}{dt}}{a^2}\rightarrow\frac{c^2}{a^2}\frac{d\theta}{dt}=\frac{a\frac{db}{dt}-b\frac{da}{dt}}{a^2}$

$$\frac{(130)^2}{(120)^2}\frac{d\theta}{dt}=\frac{(120)(5)-(50)(-10)}{(120)^2}$$

$$\frac{d\theta}{dt}=0.065\,{}^{rad}\!/_{sec}$$

17a. $t = 15 \rightarrow x = 20(15) = 300\,ft$

$r(15) = \sqrt{20^2 + 300^2} = 300.666\,ft$

17b. $r^2 = 20^2 + x^2 \rightarrow 2r\dfrac{dr}{dt} = 2x\dfrac{dx}{dt} \rightarrow 2\sqrt{20^2 + 150^2}\,\dfrac{dr}{dt} = 2(150)(20)$

$\dfrac{dr}{dt} = 19.825\ {}^{ft}\!/_{\sec}$

17c. $tan\theta = \dfrac{20}{x} \rightarrow \theta = tan^{-1}\dfrac{20}{140} = 0.142$

17d. $\sec^2\theta\dfrac{d\theta}{dt} = -20x^{-2}\dfrac{dx}{dt} \rightarrow \dfrac{d\theta}{dt} = -20\left(\dfrac{20}{140}\right)^2(15)\cos^2 0.142 = -0.015\,{}^{rad}\!/_{sec}$

19a. $A = \dfrac{1}{2}(730)(730)\sin 0.79 = 189{,}273.629\ sq.ft.$

19b. $c^2 = 370^2 + 370^2 - 2(370)(370)\cos 0.79 \rightarrow c = 284.758\,ft$

19c. $c^2 = a^2 + a^2 - 2(a)(a)\cos 0.79 \rightarrow c^2 = 0.593a^2$

$\dfrac{d}{dt}\left[c^2 = a^2 + a^2 - 2(a)(a)\cos 0.79\right] = \dfrac{d}{dt}\left[c^2 = 0.593a^2\right]$

$2c\dfrac{dc}{dt} = 1.186a\dfrac{da}{dt} \rightarrow \dfrac{dc}{dt} = \dfrac{1.186(370)}{2(284.758)}(-15.2) = -5.856\,{}^{\text{ft}}\!/_{\text{hr}}$

19d. $\frac{d}{dt}\left[A=\frac{1}{2}(\sin 0.79)a^2\right]\rightarrow\frac{dA}{dt}=(\sin 0.79)a\frac{da}{dt}=0.710(370)(-15.2)=-3995.027$ sq.ft.

21. $\frac{dC}{dt}=2\pi\frac{dr}{dt}=6\pi\rightarrow\frac{dr}{dt}=3$

Because the right triangle inside the equilateral triangle is a 30-60-90 triangle, $a=\frac{1}{2}r$

and $P=3\sqrt{3}a=\frac{3\sqrt{3}}{2}r$

$$A=\pi r^2-\frac{3\sqrt{3}}{2}r\rightarrow\frac{d}{dt}\left[A=\pi r^2-\frac{3\sqrt{3}}{2}r\right]\rightarrow\frac{dA}{dr}=2\pi r\frac{dr}{dt}-\frac{3\sqrt{3}}{2}\frac{dr}{dt}$$

$$\pi r^2=64\pi\rightarrow r=8$$

$$\frac{dA}{dr}=2\pi(8)(3)-\frac{3\sqrt{3}}{2}(3)=143.002\,{}^{in^2}\!/_{\sec}$$

Section 8–6 Multiple Choice

1. $V=s^3\rightarrow\frac{dV}{dt}=3s^2\frac{ds}{dt};\ A=6s^2=54\rightarrow s=3$

$$\frac{dV}{dt}=3(3)^2(6)=162$$

The correct answer is D

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3. $x=5(70)=350miles;\quad y=5(28)=140miles$

$$x^2+y^2=r^2\rightarrow r=\sqrt{350^2+140^2}=376.962$$

$$x^2+y^2=r^2\rightarrow 2x\frac{dx}{dt}+2y\frac{dy}{dt}=2r\frac{dr}{dt}\rightarrow x\frac{dx}{dt}+y\frac{dy}{dt}=r\frac{dr}{dt}$$

$$(350)70+(140)28=376.962\frac{dr}{dt}\rightarrow\frac{dr}{dt}=75.39$$

The correct answer is D

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5. $$V=\frac{\pi h^2}{3}(18-h)=6\pi h^2-\frac{\pi}{3}h^3\rightarrow\frac{dV}{dt}=12\pi h\frac{dh}{dt}-\pi h^2\frac{dh}{dt}.$$

$$30\pi=\left[12\pi(2)-\pi(2)^2\right]\frac{dh}{dt}\rightarrow\frac{dh}{dt}=\frac{3}{2}$$

The correct answer is C

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7. $$\frac{dx}{dt}=x;\ \frac{d}{dt}\left(\frac{1}{x}\right)=\frac{-1}{x^2}\frac{dx}{dt}=\frac{-1}{x^2}(x)=-\frac{1}{x}\therefore At\ x=-\frac{1}{4},\frac{d}{dt}\left(\frac{1}{x}\right)=4?$$

The correct answer is E