AP Calculus AB ’23-24 Name Solution Key

Spring Final Exam - Dr. Quattrin

Part IA: Multiple choice – No Calculator.
50 minutes

1. Which of the following statements is true?

a) $\int\left(\frac{10x-4}{(5x^2-4x+2)^2}\right)dx=\ln|5x^2-4x+2|^2+c$ F

b) $\int \sin^2\frac{1}{2}x\,dx=\frac{1}{4}x\cancel{+}\frac{1}{4}\sin x+c$ F

c) $\int\left(\frac{x}{\sqrt{4-x^2}}\right)dx=-\frac{1}{2}(4-x^2)^{1/2}+c$ F

d) $\int\left(\frac{e^x}{\tan e^x}\right)dx=\ln|\sin e^x|+c$ T

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2. If $f(x)=\tan^{-1}(e^{2x})$, then $f'(0)=$

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

$\frac{1}{(e^{2x})^2+1}\,e^{2x}(2) = \frac{1}{2}(1)(2)$

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3. If $\int_{-2}^{5} f(x)dx = -3$, $\int_{-2}^{1} g(x)dx = -2$, $\int_{1}^{5} f(x)dx = 5$, and $\int_{1}^{5} g(x)dx = 5$, then $\int_{5}^{-2} [2g(x) + 3f(x)]dx =$

a) -39 b) -15 c) 3 d) 5

$\int_5^{-2} = -\int_{-2}^{5} 2g - \int_{-2}^{5} 3f = -2(3) - 3(-3)$

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4. If $h(x) = 3x^2 \ln x$, then $h'(1) =$

(a) 0 (b) 1 (c) 3 (d) 9

$h'(x) = 3x^2\left(\frac{1}{x}\right) + \ln x\,(3x^2)$

$h'(1) = 3(1) + 0$

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x	1	2	4	8
$f(x)$	-3	4	9	-1
$g(x)$	0	8	2	1
$f'(x)$	2	-4	3	-2
$g'(x)$	-4	1	3	5

5. Let $h(x) = f(g(x^3))$. What is the value of $h'(2)$?

(a) 120 (b) 24 (c) 12 (d) 10

$h'(x) = f'(g(x^3)) \cdot g'(x^3) \cdot 3x^2$

$h'(2) = f'(g(8)) \cdot g'(8) \cdot (12)$

$= f'(1) \cdot g'(8)(12) = 2 \cdot 5 \cdot 12 = 1$

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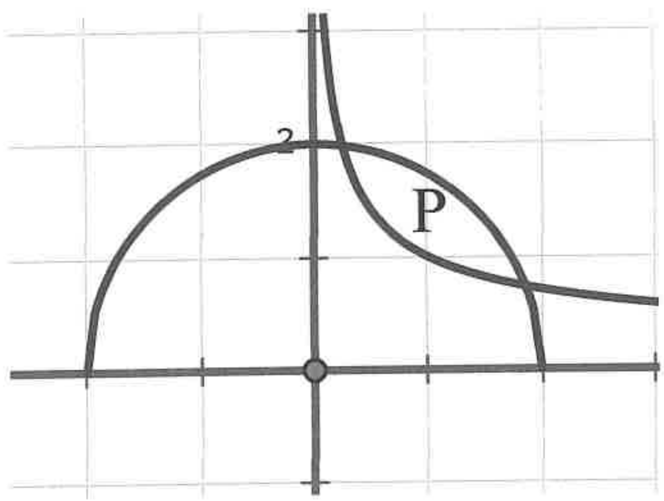
6. If $y = x\ln(2x^3)$, then $\dfrac{d^2y}{dx^2} =$ $\frac{1}{2x^3}(6x^2)$

(a) $6+\ln(2x^3)$ (b) $\dfrac{1}{2x^3}$ (c) $\dfrac{3}{x}$ (d) $\ln(2x^3)$

$\frac{dy}{dx} = x\left(\frac{1}{2x^3}(6x^2)\right) + \ln x\ (2x^3 = 3 + \ln 2x^3$



7. Region P is bounded by $y = \sqrt{4-x^2}$ and $y = \dfrac{1}{\sqrt{x}}$. The volume of the solid whose cross-sections perpendicular to the line $x = 0$ are isosceles right triangles with a leg in P is determined by

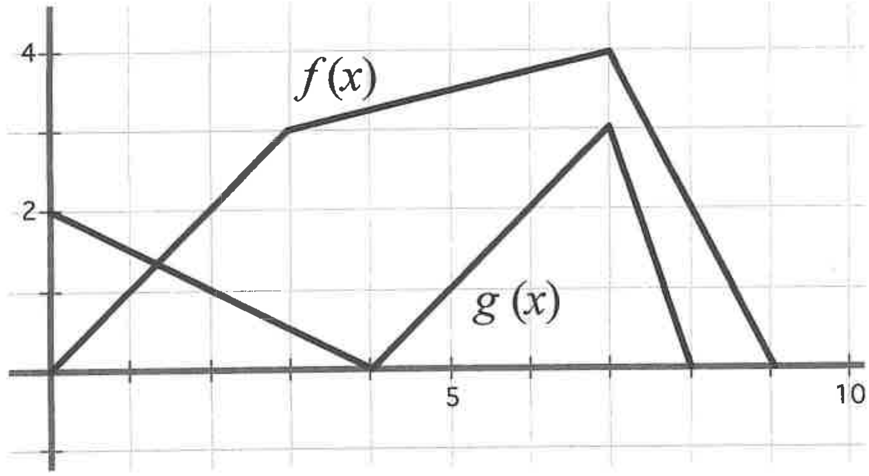
y-AXIS

(a) $\dfrac{1}{2}\displaystyle\int_{0.2}^{1.86}\left(\sqrt{4-x^2}-\frac{1}{\sqrt{x}}\right)^2 dx$ (b) $\dfrac{1}{2}\displaystyle\int_{0.733}^{2.24}\left(4-y^2-\frac{1}{y^4}\right) dy$

(c) $\dfrac{1}{2}\displaystyle\int_{0.2}^{1.86}\left(4-x^2-\frac{1}{x}\right) dx$ (d) $\dfrac{1}{2}\displaystyle\int_{0.733}^{2.24}\left(4-y^2-\frac{1}{y^2}\right)^2 dy$

(e) $\pi\displaystyle\int_{0.733}^{2.24}\left(4-y^2-\frac{1}{y^4}\right) dy$ ROTATION NO CROSS-SECTION

$\frac{1}{2}b^2$



8. Let $f(x)$ and $g(x)$ be linear differentiable functions defines by the graphs above. If $h(x) = \dfrac{g(x)}{f(x)}$, what is the value of $h'(2)$?

(a) $-\dfrac{1}{2}$ (b) 0 (c) -2 d) $\dfrac{1}{2}$

$$h'(2) = \frac{f(2)\,g'(2) - g(2)\,f'(2)}{[f(2)]^2} = \frac{2(-1/2) - 1(1)}{2^2}$$

9. $\int \left(\frac{3x^2 - x^{1/2} + 2}{\sqrt[3]{x^2}} \right) dx =$ $\int (3x^{4/3} - x^{-1/6} + 2x^{-2/3})\, dx$

(a) $\left(x^3 - \frac{2}{3}x^{3/2} + 2x\right)\left(-2x^{-1/2}\right) + c$

(b) $2x^{3/2} - \ln|x| - 4x^{-1/2} + c$

(c) $3x^{4/3} - x^{-1/6} + 2x^{-2/3} + c$

(d) $\frac{9}{7}x^{7/3} - \frac{6}{5}x^{5/6} + 6x^{1/3} + c$

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10. Consider the curve given by $y^3 - xy^2 + 4x^2 = 11$. Which of the following is true?

$3y^2 \frac{dy}{dx} - x\left(2y \frac{dy}{dx}\right) * - y^2(1) + 8x = 0$

(a) $\frac{dy}{dx} = \frac{y^2 - 8x}{3y^2 - 2xy}$

(b) $\frac{dy}{dx} = \frac{8x - y^2}{3y^2 - 2xy}$

(c) $\frac{dy}{dx} = \frac{11 + y^2 - 8x}{3y^2 - 2xy}$

(d) $\frac{dy}{dx} = \frac{-8x}{3y^2 - 2xy}$

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11. A ski resort uses a snow machine to control the snow level on a ski slope. Over a 24-hour period the volume of snow added to the slope per hour is modeled by the equation $S(t)$. The rate that the snow melts is modeled by $M(t)$. Both $M(t)$ and $S(t)$ are measured in $\frac{yd^3}{h}$ and t is measured in hours for $0 \le t \le 24$. At time $t = 0$, the slope holds $50yd^3$ of snow. Which of the following expresses the total amount of snow on the ground?

(a) $\int_0^t [S(x) - M(x)]\, dx$

(b) $\frac{1}{t} \int_0^t [S(x) - M(x)]\, dx$

(c) $S(t) - M(t)$

(d) $S'(t) - M'(t)$

(e) $50 + \int_0^t [S(x) - M(x)]\, dx$

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12. This problem involves finding the absolute maximum and absolute minimum of the function $f(x) = x^4 + 8x^2 + 1$ restricted to the closed interval $x \in [-3, 1]$. Which of the following statements is correct?

(a) $f(x)$ has both an absolute maximum and absolute minimum at an end point.
(b) $f(x)$ has both an absolute maximum and absolute minimum at interior points.
(c) $f(x)$ has both an absolute maximum at an end point and an absolute minimum at an interior point.
(d) $f(x)$ has both an absolute maximum at an interior point and an absolute minimum at an end point.

$f'(x) = 4x^3 + 16x^2 = 4x(x-4) = 0$

$x > 0, \cancel{4}$

x	y
-3	>10
0	1
1	10

13. The acceleration of a particle is described by $a(t) = e^t - \sin 2t$. Find the distance equation for $x(t)$ if $v(0) = 0$ and $x(0) = 3$.

(a) $x(t) = e^t + \sin 2t$

(b) $x(t) = e^t - \frac{1}{2}\cos 2t - \frac{3}{2}$

(c) $x(t) = e^t + \frac{1}{2}\sin 2t - \frac{3}{2}t + 2$

(d) $x(t) = e^t + \frac{1}{4}\sin 2t - \frac{3}{2}t + 2$

$V = \int (e^t - \sin 2t)\,dt$

$= e^t + \frac{1}{2}\cos 2t + C_1$

$(0,0) \Rightarrow 1 + \frac{1}{2} + C_1$

$\frac{-3}{2} = C_1$

$X = \int e^t + \frac{1}{2}\cos 2t - \frac{3}{2}\,dt$

14. Suppose f is a differentiable function such that $f(-1)=2$ and $f'(-1)=-3$. Using the line tangent to the graph of $f(x)$ at $x=-1$, find the approximation of $f(-0.9)$.

$y-2=-3(x+1)$

(a) 1.7 (b) -2.3 (c) -0.03 (d) 7.7

$y(-0.09) = 2+(-.3)$

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15. Given the function $h(x)$ which is differentiable at $x=2$, find the values of m and k.

$$h(x)=\begin{cases} mx+\ln(x-1)^k, & \text{if } x>2 \\ mx^2-5x, & \text{if } x\le 2 \end{cases}$$

~~(a)~~ $m=-10,\ k=-5$ ~~(b)~~ $m=-5,\ k=-10$

(c) $m=5,\ k=10$ ~~(d)~~ $m=10,\ k=5$

$x=2 \rightarrow 2m+0=\cancel{8}\cdot 4m-10$

$10=2m$

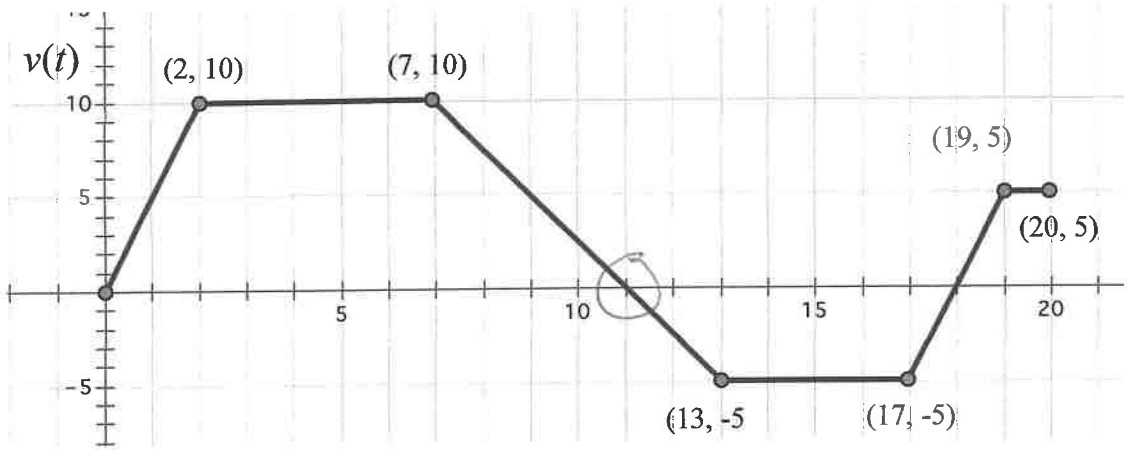
$5=m$

$h'(x)=\begin{cases} m+\frac{k}{x-1} \\ 2mx-5 \end{cases}$

$5+\frac{k}{1}=20-5=15$

$\frac{k}{1}=10$

16. A couple take their new dog Skadi to run around at Fort Funston. She immediately runs away and back toward them several times. For $0 \le t \le 20$, Skadi's velocity is modeled by the piecewise-linear function defined by the graph below:



where $v(t)$ is measured in feet per second and t is measured in seconds.
At $t = 11$, Skadi is

a) Speeding up.
b) Slowing down.
c) Neither speeding up nor slowing down.
d) Speeding up or slowing down cannot be determined.

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17. As SI begins the New Learning Commons Project Campaign, they find they need to raise \$200 million dollars. They hope to raise the money so that they can retire the debt in five years. The rate at which donations F, in millions of dollars per month, needs to receive in order to realize the goal in 5 years would be expressed by $\dfrac{dF}{dt} = 1.842\left(50 - \dfrac{F}{4}\right)$. One rule of thumb for capital campaigns is that you should have half the money to complete the project before you begin—which means that $F(0) = 100$. Find $\lim_{t \to \infty} F(t)$.

$\left(50 - \frac{F}{4}\right) = \frac{1}{4}(200 - F)$

(a) 4 (b) 50 (c) 100 (d) 200 (e) dne

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18. The puffin population on the Skellig Islands off the coast of County Kerry, Ireland, can be modeled by a differentiable function P in terms of time t, where $P(t)$ is the number of puffins and t is measured in years, for $0 \le t \le 50$. There are 10,000 puffins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

$$B(t) = 500e^{0.05t} \text{ puffins per year}$$

and the death rate for the penguins on the island is modeled by

$$D(t) = 110e^{0.09t} \text{ puffins per year}$$

What would the units be for $\frac{1}{50}\int_0^{50}\left(500e^{0.05t} - 110e^{0.09t}\right)dt$? $= \frac{1}{\text{year}} \cdot$ PUFFINS

(a) Puffins

(b) Puffins per year

(c) Puffins per year per year

(d) Puffins - years

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19. Which of the following is the solution to the differential equation

$\frac{dy}{dx} = \frac{x-1}{y}$ with the initial condition $y(0) = -2$?

~~(a)~~ $y = -2e^{x^2-2x}$

~~(b)~~ $y = -2 + e^{x^2-2x}$

(c) $y = \sqrt{x^2-2x-4}$

(d) $y = -\sqrt{x^2-2x+4}$

(e) $y = -\sqrt{x^2-2x-4}$

$y\,dy = (x-1)\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} - x + c$

$(0,-2) \to \frac{4}{2} \ldots$ $2 = c$

$\frac{y^2}{2} = \frac{x^2}{2} - 2x \ldots$

$y^2 = x^2 - 2x + 4$

20. $\int_0^1 (32z-2)\sqrt[3]{8z^2-z+1}\,dz =$

$u = 8z^2 - z + 1$

$du = (16z - 1)\,dz$

(a) 45 (b) 27.5 (c) 12 (d) 4.5

$\int_0^1 = 2\int_1^8 u^{1/3}\,du$

$= 2\left.\frac{u^{4/3}}{4/3}\right]_1^8$

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21. Suppose $f'(x) = \dfrac{(x+1)^3(x-4)^4}{(x^2+4)}$. Of the following, which best describes the graph of $f(x)$?

f' − 0 + 0 +

x −1 4

(a) $f(x)$ has relative minimum at $x = -1$ and a relative maximum at $x = 4$.

(b) $f(x)$ has relative maximum at $x = -1$ and a relative minimum at $x = 4$.

(c) $f(x)$ has relative minimum at $x = -1$ and a point of inflection at $x = 4$.

(d) $f(x)$ has relative maximum at $x = -1$ and a point of inflection at $x = 4$.

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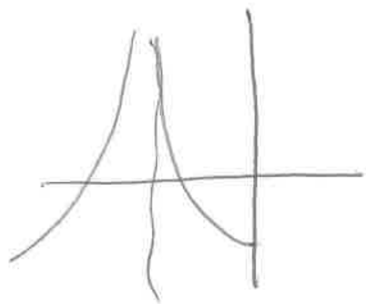
22. A function $f(x)$ has a vertical asymptote at $x = -2$. The derivative of $f(x)$ is positive for all $x < -2$ and negative for all $-2 < x$. Which of the following statements is **true**?

a) $\lim\limits_{x \to -2^-} f(x) = -\infty$ and $\lim\limits_{x \to -2^+} f(x) = -\infty$

b) $\lim\limits_{x \to -2^-} f(x) = -\infty$ and $\lim\limits_{x \to -2^+} f(x) = +\infty$

c) $\lim\limits_{x \to -2^-} f(x) = +\infty$ and $\lim\limits_{x \to -2^+} f(x) = +\infty$

d) $\lim\limits_{x \to -2^-} f(x) = +\infty$ and $\lim\limits_{x \to -2^+} f(x) = -\infty$



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23. $$\lim_{x \to 0} \frac{\int_{2x}^{0} e^t\,dt}{\sin x} = \overset{L'H}{} \lim_{x \to 0} \frac{-e^{2x}(2)}{\cos x}$$

(a) 0 (b) 1 (c) -1 (d) 2 (e) -2

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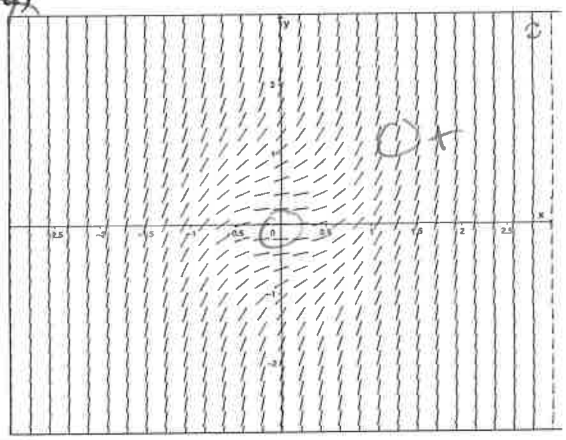
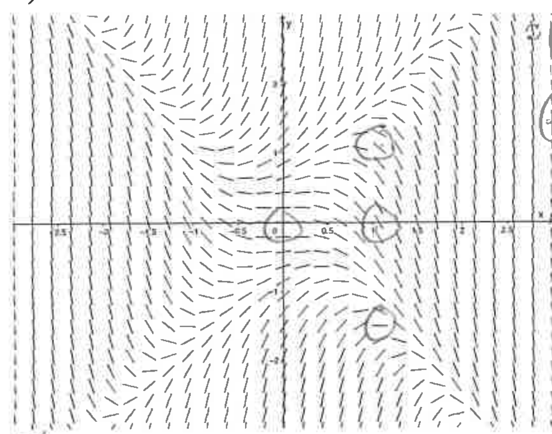
24. Which of the following slope fields represents $\frac{dy}{dx} = y^2 - 2x^2$?

x, y	m
(0, 0)	0
(1, 0)	−1
(1, 1)	−1
(−1, −1)	−1

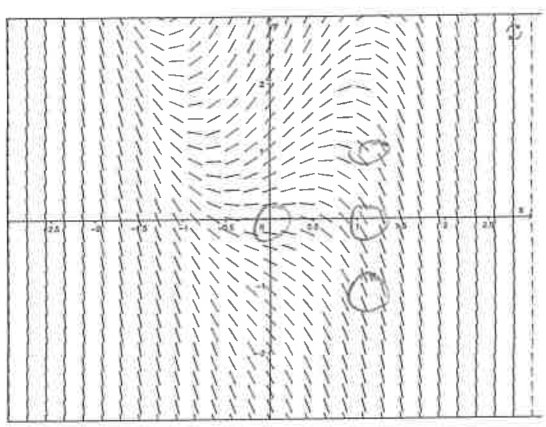
a)



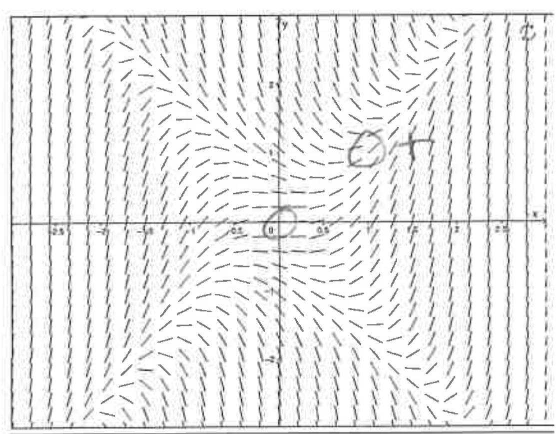
b)

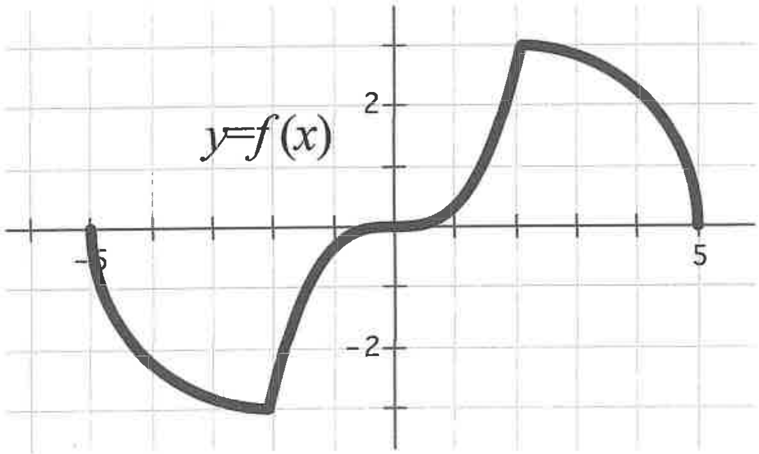


c)



d)





25. The graph of the function $f(x)$ is shown above. If $h(x)=\int_{-5}^{x} f(t)\,dt$, on what interval is $h(x)$ increasing and concave down?

a) (-5, -2) b) (-2, 0) c) (0, 2) d) (2, 5)

$h' = f$ is + & Dec

AP Calculus AB '23-24
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Name SOLUTION

Part IB: Multiple choice – Calculator allowed.
45 minutes

1. A function $f(x)$ is defined such that $f'(t) = 2.6\sqrt{x}\cos x - \dfrac{e^{0.5x}}{x+4}$ on $0 \le x \le 6$. On what interval(s) is $f(x)$ concave down?

(a) $0 \le x \le 1.449$ (b) $1.449 \le x \le 4.943$

(c) $0.635 \le x \le 3.345$ (d) $3.345 \le x \le 6$

GRAPH ON CALC &
LOOK FOR DECREASING

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2. At $x = 0$, the function given by $f(x) = \begin{cases} \sec x, & \text{if } x < 0 \\ 1, & \text{if } x = 0 \\ 1 - x^3, & \text{if } x > 0 \end{cases}$ is

$f'(x) = \begin{cases} \sec x \tan x \\ -3x^2 \end{cases}$

(a) Continuous but not differentiable

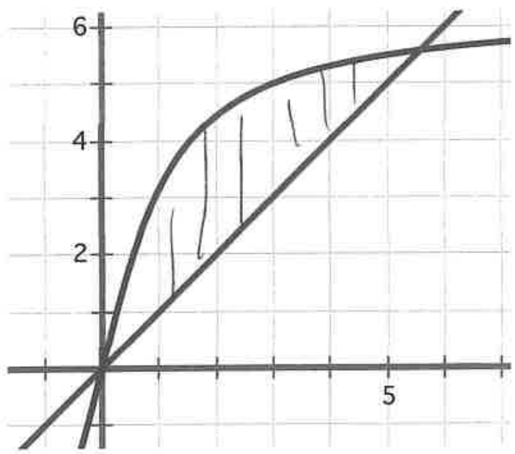
(b) Differentiable but not continuous

(c) Neither continuous nor differentiable

(d) Both continuous and differentiable

$\sec 0 = 1 = 1 - 0$ CONT

$\sec 0 \tan 0 = 0 = -3(0)^2$



$$\pi \int_0^{5.573} (\tan^{-1} x)^2 - x^2 \, dx$$

3. The region in the first quadrant enclosed by the graphs of $y = 4\tan^{-1}x$ and $y = x$ is revolved about the x-axis. The volume of this solid is

(a) 5.573 (b) 180.398 (c) 128.732 (d) 8.594

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4. Let $V(t)$ and $r(t)$ denote the volume and radius, respectively, of the icicle t hours after 10:00 a.m. Assume that the icicle continued to melt from $t = 0$ (10:00 a.m.) to $t = M$. Which of the following statements below that must be true if "After the icicle began dripping at 10:00 a.m., it took exactly M hours for the icicle to melt completely"?

(a) $\int_0^M V'(t)dt > \int_0^{\frac{M}{2}} V'(t)dt$ (b) $\int_0^M V'(t)dt = -V(0)$

(c) $\int_0^M V'(t)dt = 0$ (d) $\int_0^M r(t)dt = -2$

$$\int_0^M V' = \text{VOLUME} = V(M) - V(0)$$
$$= 0 - V(0)$$

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5. What is the average rate of change for the function $y = \csc\left(\frac{x}{2}\right)$ from $x = \frac{\pi}{2}$ to $x = \pi$?

(a) -0.707 (b) -0.263 (c) 0.450 (d) 1.122 (e) 1.763

$$\frac{y(\pi) - y(\pi/2)}{\pi - \pi/2} = \frac{\phi - \sqrt{2}}{\pi/2}$$

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6. A particle is moving along the x-axis so that its velocity is given by $v(t) = \dfrac{9\ln(t)\left(\frac{t^2}{4}\right)}{t^2+1}$. What is the average distance traveled on $t \in [0, 6]$.

(a) 3.0 (b) 2.018 (c) 1.955 (d) 0.228

$$\text{Ave}_{\text{Dist}} = \frac{1}{6}\int_0^6 |v(t)|$$

7. Given the functions $f(x)$ and $g(x)$ that are both continuous and differentiable, and that they have values given on the table below.

x	$f'(x)$	$f''(x)$	$g'(x)$	$g''(x)$
2	-1	2	-8	-5
4	8	-11	4	3
8	-3	-12	-1	4

At what value of x is $g(x)$ increasing and concave down?

(a) $x=2$ (b) $x=4$ (c) $x=8$ (d) None of these

$g'>0 \quad g''<0$

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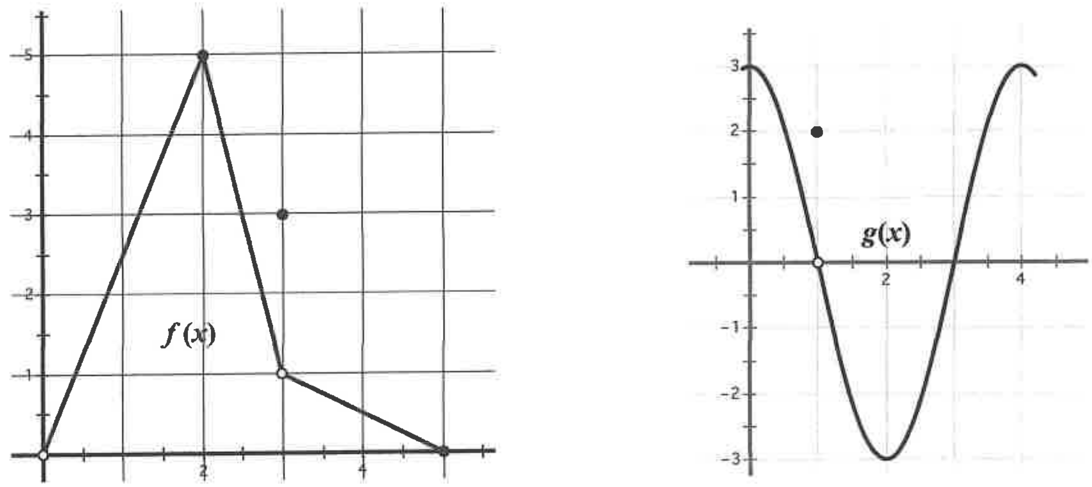
8. Let $f(x)=\begin{cases} \sqrt[3]{k^2}+2x, & if\ x\le 2 \\ \dfrac{4}{k^2-x^2}, & if\ 2<x \end{cases}$. Which of the following values of k would make $y=f(x)$ continuous?

a) 2.169 b) 2.136 c) 2.838 (d) 5.676

$k^{2/3}+4=\dfrac{4}{k^2-4}$ GRAPH TO FIND INTERSECTION



9. Which of the following statements is true?

$g\left(\lim_{x\to 3} f(x)\right) = f(\ )$

(a) $\lim_{x \to 1} f(g(x)) = 5$ F

(b) $\lim_{x \to 3} g(f(x)) = 2$ T

(c) $\lim_{x \to 2} \left[\frac{g(x)}{f(x+1)}\right] = -1$

(d) $\lim_{x \to 2} \left[\frac{f(x+1)}{g(x-2)}\right] = 1$

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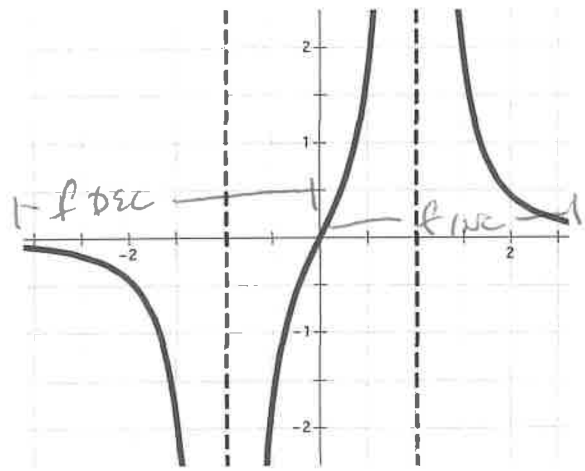
10. If $f(x) = \begin{cases} e^{2x} - 1, if x \le 0 \\ 1 - \cos x, if 0 < x \end{cases}$, then $\int_{-1}^{1} f(x)dx =$

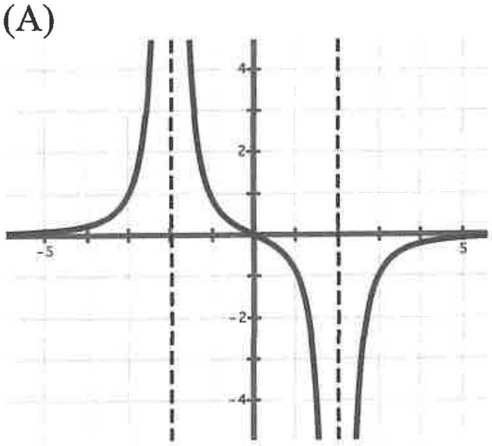
(a) -0.409 (b) 1.310 (c) 1.944 (d) 2.353

$\int_{-1}^{0}(e^{2x}-1)\,dx + \int_{0}^{1}(1-\cos x)\,dx =$

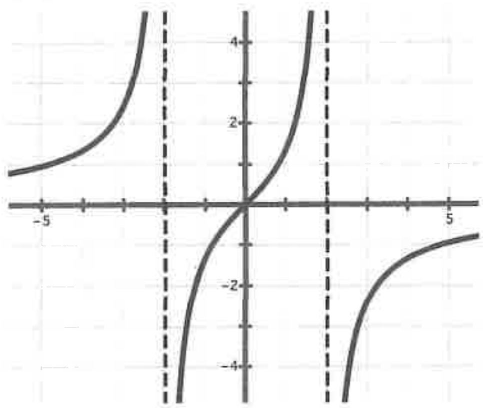


11. The graph of $f'(x)$ is shown above. Which graph below is most likely to be $f(x)$?

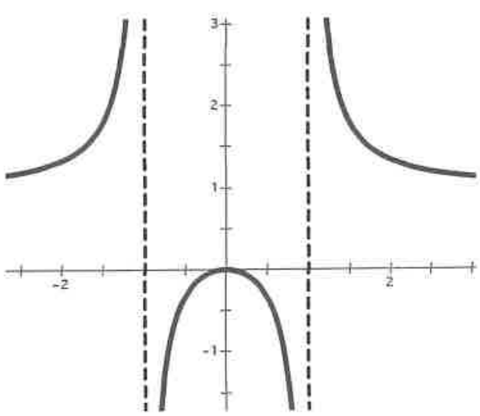
(A)



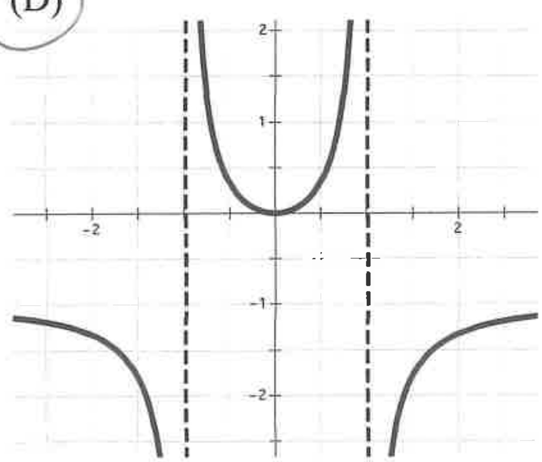
(B)



(C)



(D)



12. If $g(x) = 4 + \int_{2}^{x} \left(3 - \sqrt{x^2 - x + 3\sin 2x}\right) dx$, then the x-value of the relative minimum of $g(x)$ is

$g' = 0$ & CHANGES FROM − TO +

(a) -2.085 (b) 0.843 (c) 1.915 (d) 3.342

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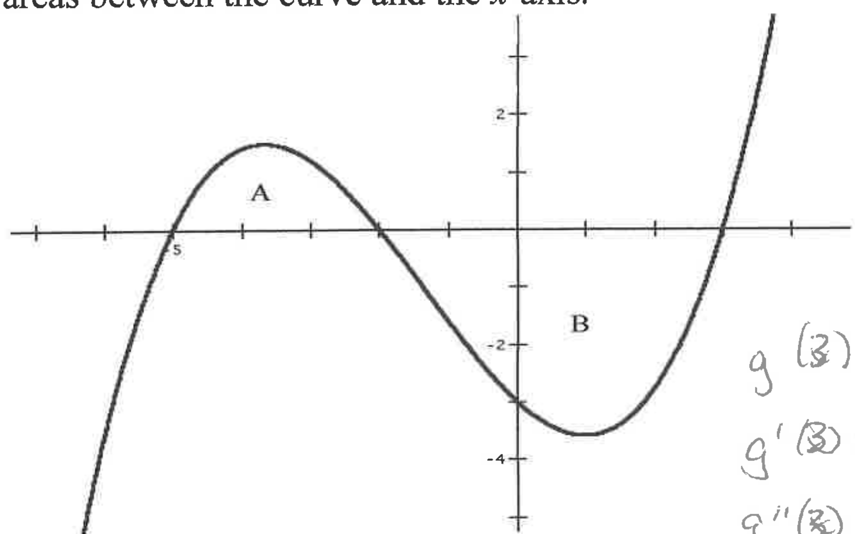
13. The graph of $y = f(x)$ is shown below. A and B are positive numbers that represent the areas between the curve and the x-axis.



$g(3) < 0$ BECAUSE $B > A$

$g'(3) = f(3) = 0$

$g''(3) = f'(3) > 0$

If $g(x) = \int_{-5}^{x} f(t)\,dt$, which of the following must be true?

(a) $g(3) < g'(3) < g''(3)$ (b) $g'(3) < g(3) < g''(3)$

(c) $g(3) < g''(3) < g'(3)$ (d) $g''(3) < g(3) < g'(3)$

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14. $\lim\limits_{x \to 1} \dfrac{\sqrt{x^2-1}}{1-x} \overset{LH}{=} \lim\limits_{x \to 1} \dfrac{\frac{x}{\sqrt{x^2-1}}}{-1} = \dfrac{-1}{0}$

(a) 0 (b) 1 (c) -1 (d) DNE

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x	2	5	10	14
$f(x)$	12	28	34	30

15. Let f be a differentiable function on the closed interval [2, 14] and which has values as shown on the table above. Using the sub-intervals defined by the table values and using right-hand Riemann sums, $\int_2^{14} f(x)\,dx =$

(a) 296 (b) 312 (c) 343 (d) 374 (e) 390

$3(28)+5(34)+4(30)$
