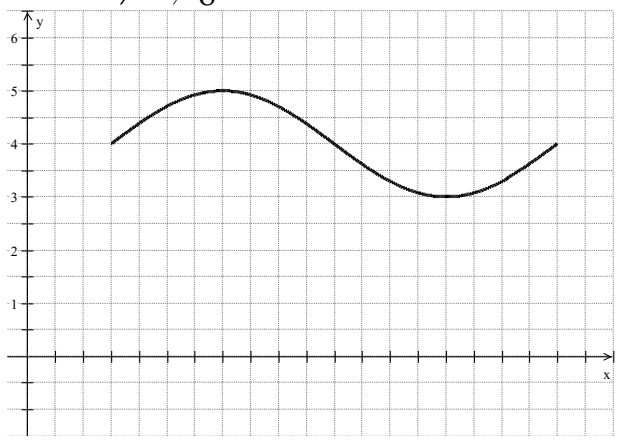
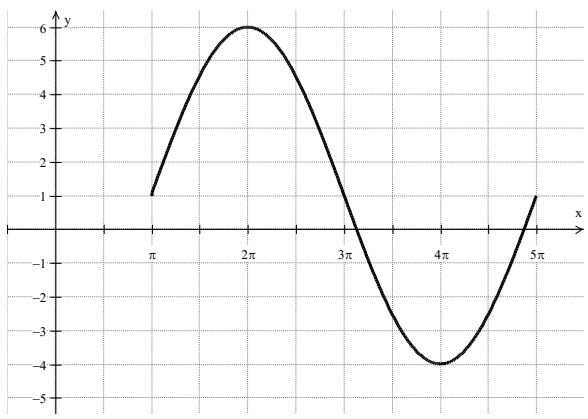
Parent Trigonometric Functions Odd Solutions

2-1 Free Response Homework

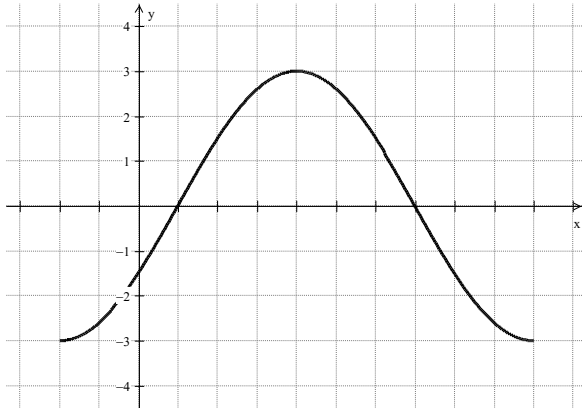
1. $A = 1$ Period = $\dfrac{2\pi}{\pi/8} = 16$ ___ $k = 4$ $h = 3$



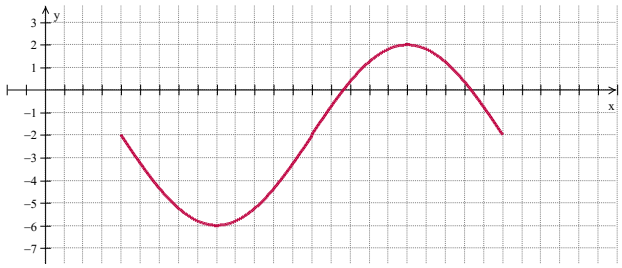
3. $A = 5$ Period = $\dfrac{2\pi}{1/2} = 4\pi$ $k = 1$ $h = \pi$



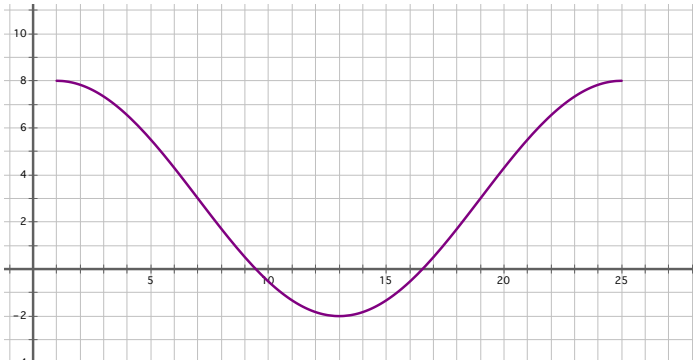
5. $A = 3$ Period $= \dfrac{2\pi}{\pi/6} = 12$ $k = 0$ $h = -2$



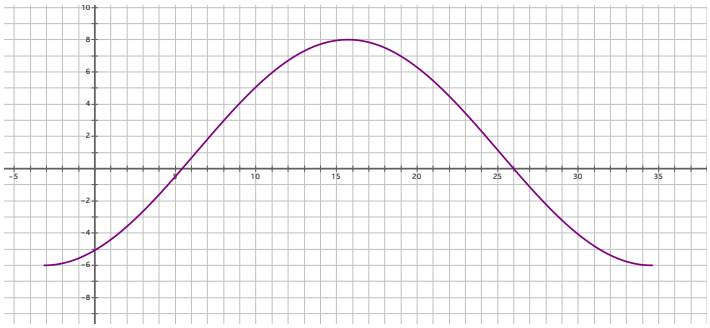
7. $A = 4$ Period $= \dfrac{2\pi}{\pi/5} = 10$ $k = -2$ $h = 2$



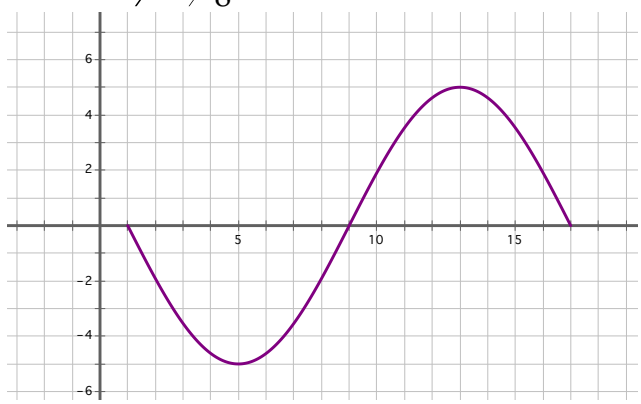
9. $A = 3$ Period $= \dfrac{2\pi}{\pi/12} = 24$ $k = 3$ $h = 1$



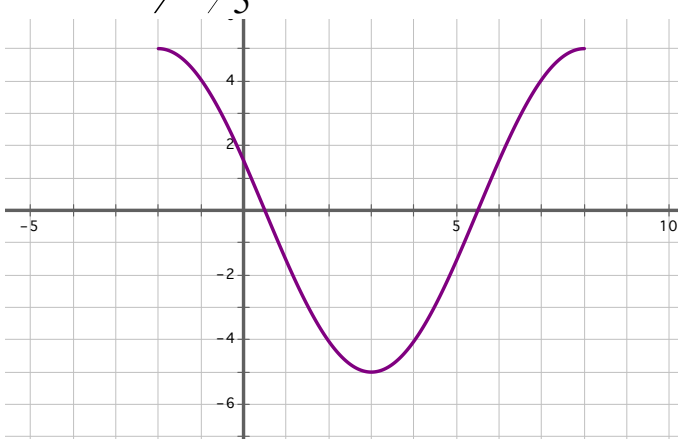
11. $A = 1$ Period $= \dfrac{2\pi}{1/6} = 12\pi$ $k = 1$ $h = -\pi$



13. $A = 2$ Period $= \dfrac{2\pi}{\pi/8} = 16$ $k = 0$ $h = 1$



15. $A = 5$ Period $= \dfrac{2\pi}{\pi/5} = 10$ $k = 0$ $h = -2$



17a. With $A = 70$ and $k = -4$
minimum value @ $T = -110°\text{F}$; maximum value @ $T = 30°\text{F}$

True

b. False Period = $2\pi/B = 2\pi \Big/ \pi/7 = 14$

c. False

$$T(0) = -40 + 70\sin\left[\left(\pi/7\right)(0-5)\right] = -94.728°\text{F}$$

d. True

$$-10 = -40 + 70\sin\left[\pi/7\,(x-5)\right]$$

$$\sin^{-1}\left(\frac{30}{70}\right) = \pi/7\,(x-5)$$

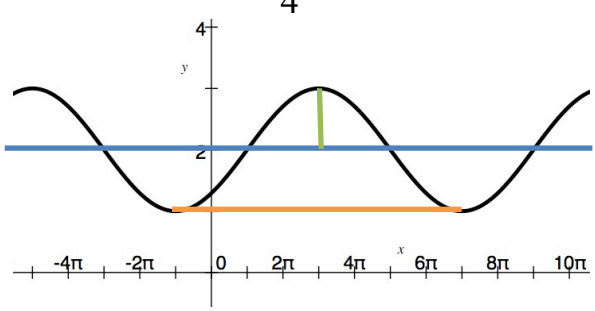
$$\pi/7\,(x-5) = \begin{cases} 0.443 \pm 2\pi n \\ 2.699 \pm 2\pi n \end{cases}$$

$$x - 5 = \begin{cases} 0.989 \pm 14n \\ 6.013 \pm 14n \end{cases}$$

$$x = \begin{cases} 5.989 \pm 14n \\ 11.013 \pm 14n \end{cases}$$

$$x = 5.989,\ 11.013$$

19. $y = 2 + \cos\frac{1}{4}(x - 3\pi)$ $\quad$ $y = 2 + \sin\frac{1}{4}(x - \pi)$



Minimum y value = 1 $\qquad$ Maximum y value = 3

$$\Rightarrow k = \frac{1+3}{2} = 2$$

$$\Rightarrow \text{Amplitude} = 3 - 2 = 2 - 1 = 1$$

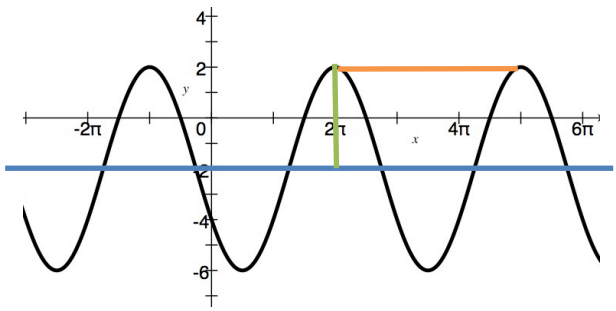
Period $= 7\pi - (-\pi) = 8\pi$

$\Rightarrow B = \dfrac{2\pi}{8\pi} = \dfrac{1}{4}$

$h = 3\pi$ for cosine (not a unique answer)

$h = \pi$ for sine (not a unique answer)

21. $y = -2 + 4\cos\frac{2}{3}(x - 2\pi)$ $\qquad$ $y = -2 + 4\sin\frac{2}{3}\left(x - \frac{5\pi}{4}\right)$



Minimum y value = –6 $\qquad$ Maximum y value = 2

$\Rightarrow k = \dfrac{-6+2}{2} = -2 \Rightarrow$ Amplitude $= 2 - (-2) = -2 - (-6) = 4$

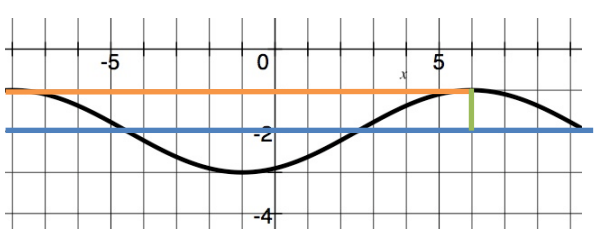
Period $= 5\pi - 2\pi = 3\pi$

$\Rightarrow B = \dfrac{2\pi}{3\pi} = \dfrac{2}{3}$

$h = 2\pi$ for cosine (not a unique answer)

$h = 2\pi - \dfrac{3\pi}{4} = \dfrac{5\pi}{4}$ for sine (not a unique answer)

23. $y = -2 + \cos\frac{\pi}{7}(x - 6)$ $\qquad$ $y = -2 + \sin\frac{\pi}{7}\left(x - \frac{5}{2}\right)$



Minimum y value = –3 $\qquad$ Maximum y value = –1

$$\Rightarrow k = \frac{-3+(-1)}{2} = -2$$

$$\Rightarrow \text{Amplitude} = -1-(-2) = -2-(-3) = 1$$

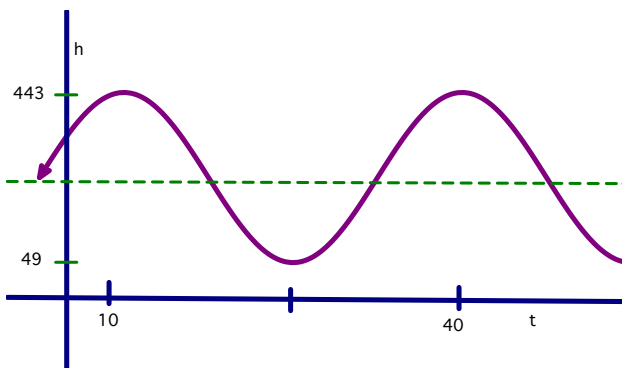
$$\text{Period } = 6-(-8) = 14$$

$$\Rightarrow B = \frac{2\pi}{14} = \frac{\pi}{7}$$

$h = 6$ for cosine (not a unique answer)

$h = 6 - \frac{14}{4} = \frac{5}{2}$ for sine (not a unique answer)

25. $h = 246 + 197\cos\left(\frac{\pi}{15}(t-10)\right)$ $\quad h = 246 + 197\sin\left(\frac{\pi}{15}(t-3.5)\right)$



Minimum y value = –49 $\qquad$ Maximum y value = 443

$$\Rightarrow k = \frac{443+49}{2} = 246$$

$$\text{Amplitude} = 443 - 246 = 197$$

$$\text{Period } = 40 - 10 = 30$$

$$\Rightarrow B = \frac{2\pi}{30} = \frac{\pi}{15}$$

$h = 10$ for cosine (not a unique answer)

$h = 10 - \frac{30}{4} = \frac{7}{2}$ for sine (not a unique answer)

2-1 Multiple Choice Homework

1. E $y = 2\sin 2x$

Amplitude = 2, $period = \pi \therefore B = \dfrac{2\pi}{\pi} = 2$

3. E

$$\cos 100° = -0.174 \rightarrow \sin^{-1}(-0.174) = -10° = -10 \bullet \frac{\pi}{180} = -0.175 \approx -0.2$$

5. B

$$2t - 40° = \cos^{-1}(1) = 0$$
$$2t = 40°$$
$$t = 20°$$

7. B $y = 4 - 3\cos\left[\dfrac{1}{4}\left(x - \dfrac{\pi}{2}\right)\right]$

$$k = 4;\ A = 3;\ half - period = 4\pi \therefore B = \frac{\pi}{4\pi} = \frac{1}{4};\quad h = \frac{\pi}{2} \therefore A < 0$$

2-2 Free Response Homework

1a. $y(-2.7) = 2 + 3\cos\left[\dfrac{\pi}{4}(-2.7)\right] = 0.433$

b. $x \in \{2.929,\ 5.071,\ 10.929\}$

$$2 + 3\cos\left(\frac{\pi}{4}x\right) = 0$$
$$\cos\left(\frac{\pi}{4}x\right) = -\frac{2}{3}$$

$$\frac{\pi}{4}x = \begin{cases} 2.301 \pm 2\pi n \\ -2.301 \pm 2\pi n \end{cases} = \pm 2.301 \pm 2\pi n$$

$$x = \begin{cases} 2.929 \pm 8n \\ -2.929 \pm 8n \end{cases} = \pm 2.929 \pm 8n$$

$$x \in \{2.929,\ 5.071,\ 10.929\}$$

3a. $y(-2.7) = 1 + 5\sin\left[\frac{1}{2}(-2.7 - \pi)\right] = -0.095$

b. $x \in \{2.739, 9.827, 15.305\}$ 2.739, 9.827, 15.305

$$1 + 5\sin\left[\frac{1}{2}(x - \pi)\right] = 0$$

$$\sin\left[\frac{1}{2}(x - \pi)\right] = -\frac{1}{5}$$

$$\frac{1}{2}(x - \pi) = \begin{cases} -0.201 \pm 2\pi n \\ 3.343 \pm 2\pi n \end{cases}$$

$$x - \pi = \begin{cases} -0.403 \pm 4\pi n \\ 6.686 \pm 4\pi n \end{cases}$$

$$x = \begin{cases} 2.739 \pm 4\pi n \\ 9.827 \pm 4\pi n \end{cases}$$

$$x \in \{2.739, 9.827, 15.305\}$$

5a. $y(4.8) = -3\cos\left[\frac{\pi}{6}(4.8 + 21)\right] = -1.763$

b. $x \in \{-24, -18, -12, -6\}$

$$-3\cos\left[\frac{\pi}{6}(x + 21)\right] = 0$$

$$\cos\left[\frac{\pi}{6}(x + 21)\right] = 0$$

$$\frac{\pi}{6}(x+21)=\begin{cases}\frac{\pi}{2}\pm 2\pi n\\ -\frac{\pi}{2}\pm 2\pi n\end{cases}=\pm\frac{\pi}{2}\pm 2\pi n$$

$$x+21=\begin{cases}3\pm 12n\\ -3\pm 12n\end{cases}=\pm 3\pm 12n$$

$$x=\begin{cases}-18\pm 12n\\ -24\pm 12n\end{cases}$$

$$x\in\{-24,-18,-12,-6\}$$

7a. $y(4.8)=0.7+\sin\left[\frac{\pi}{8}(4.8-3)\right]=1.349$

b. $x\in\{-3.025,-14.975,-19.025,-30.975\}$

$$0.7+\sin\left[\frac{\pi}{8}(x-3)\right]=0$$

$$\sin\left[\frac{\pi}{8}(x-3)\right]=-0.7$$

$$\frac{\pi}{8}(x-3)=\begin{cases}-0.775\pm 2\pi n\\ 3.917\pm 2\pi n\end{cases}$$

$$x-3=\begin{cases}-1.975\pm 16n\\ 9.975\pm 16n\end{cases}$$

$$x=\begin{cases}1.025\pm 16n\\ 12.975\pm 16n\end{cases}$$

$$x\in\{-3.025,-14.975,-19.025,-30.975\}$$

9. $x\in\{6.863, 11.987, 19.429, 24.553\}$

$$3+7\cos\left[\frac{2\pi}{5}\left(x-11\right)\right]=5$$

$$\cos\left[\frac{1}{2}\left(x-\pi\right)\right]=\frac{2}{7}$$

$$\frac{1}{2}\left(x-\pi\right)=\begin{cases}1.281\pm 2\pi n\\ -1.281\pm 2\pi n\end{cases}$$

$$x-\pi=\begin{cases}2.562\pm 4\pi n\\ -2.562\pm 4\pi n\end{cases}$$

$$x=\begin{cases}-0.580\pm 4\pi n\\ -5.704\pm 4\pi n\end{cases}$$

On $0\le x\le 25,\ x\in\left\{6.863, 11.987, 19.429, 24.553\right\}$

11. $x \in \{-0.327, -1.007, -2.993, -3.673\}$

$$2-5\cos\left[\frac{3\pi}{4}(x-2)\right]=-1.2$$

$$\cos\left[\frac{3\pi}{4}(x-2)\right]=-0.64$$

$$\frac{3\pi}{4}(x-2)=\begin{cases} 0.802\pm 2\pi n \\ -0.802\pm 2\pi n \end{cases}$$

$$x-2=\begin{cases} 0.340\pm\frac{8}{3}n \\ -0.340\pm\frac{8}{3}n \end{cases}$$

$$x=\begin{cases} 2.340\pm\frac{8}{3}n \\ 1.660\pm\frac{8}{3}n \end{cases}$$

On $-5\le x\le 0$, $x\in\{-0.327, -1.007, -2.993, -3.673\}$.

13. $x \in \{6.325, 25.093, 31.458, 50.225\}$

$$1-4\cos\left[\frac{1}{4}(x-\pi)\right]=-1.8$$

$$\cos\left[\frac{1}{4}(x-\pi)\right]=0.7$$

$$\frac{1}{4}(x-\pi)=\begin{cases} 0.795\pm 2\pi n \\ -0.795\pm 2\pi n \end{cases}$$

$$x-\pi=\begin{cases} 3.182\pm 8\pi n \\ -3.182\pm 8\pi n \end{cases}$$

$$x=\begin{cases} 6.325\pm 8\pi n \\ -0.040\pm 8\pi n \end{cases}$$

On $0\le x\le 51$, $x \in \{6.325, 25.093, 31.458, 50.225\}$.

15. $x \in \{-29.706, -48.836, -79.972, -99.102\}$

$$1+3\sin\left[\frac{1}{8}\left(x+\frac{\pi}{2}\right)\right]=2.1$$

$$\sin\left[\frac{1}{8}\left(x+\frac{\pi}{2}\right)\right]=0.367$$

$$\frac{1}{8}\left(x+\frac{\pi}{2}\right)=\begin{cases} 0.375\pm 2\pi n \\ 2.766\pm 2\pi n \end{cases}$$

$$x+\frac{\pi}{2}=\begin{cases} 3.000\pm 16\pi n \\ 22.129\pm 16\pi n \end{cases}$$

$$x=\begin{cases} 1.429\pm 16\pi n \\ 20.559\pm 16\pi n \end{cases}$$

On $-100\le x\le 0$, $x \in \{-29.706, -48.836, -79.972, -99.102\}$

2-2 Multiple Choice Homework

1. C

$$\sin 3\left(x+\frac{2\pi}{3}\right)=0$$

$$3\sin\left(2x+\frac{4\pi}{3}\right)=0$$

$$2x+\frac{4\pi}{3}=\begin{cases}0\pm 2\pi n\\ \pi\pm 2\pi n\end{cases}=0\pm\pi n$$

$$2x=\begin{cases}-\frac{4\pi}{3}\pm 2\pi n\\ -\frac{\pi}{3}\pm 2\pi n\end{cases}=-\frac{4\pi}{3}\pm\pi n$$

$$x=\begin{cases}-\frac{2\pi}{3}\pm\pi n\\ -\frac{\pi}{6}\pm\pi n\end{cases}=-\frac{2\pi}{3}\pm\frac{\pi}{2}n$$

$$x=-2.094,-0.527,\underline{1.047},...$$

3. C

y values get larger, then smaller as x values increase (i.e. move left to right)

5. B

$$\sin 3\left(x+\frac{2\pi}{3}\right)=0$$

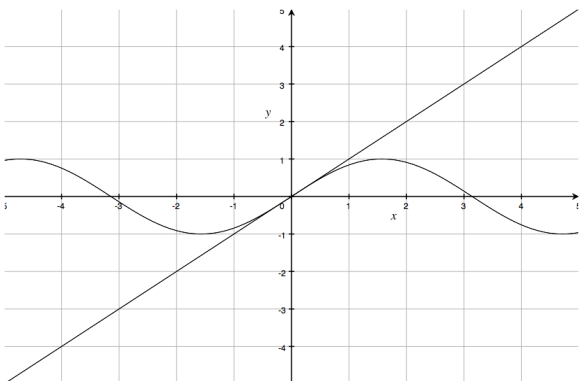
$$\sin 3x=0$$

$$3x=\begin{cases} 0\pm 2\pi n \\ \pi\pm 2\pi n \end{cases} =0\pm\pi n$$

$$x=\begin{cases} 0\pm\frac{2\pi}{3}n \\ \frac{\pi}{3}\pm\frac{2\pi}{3}n \end{cases} =0\pm\frac{\pi}{3}n$$

$$x=0,\frac{\pi}{3},\underline{\frac{2\pi}{3}},\ldots$$

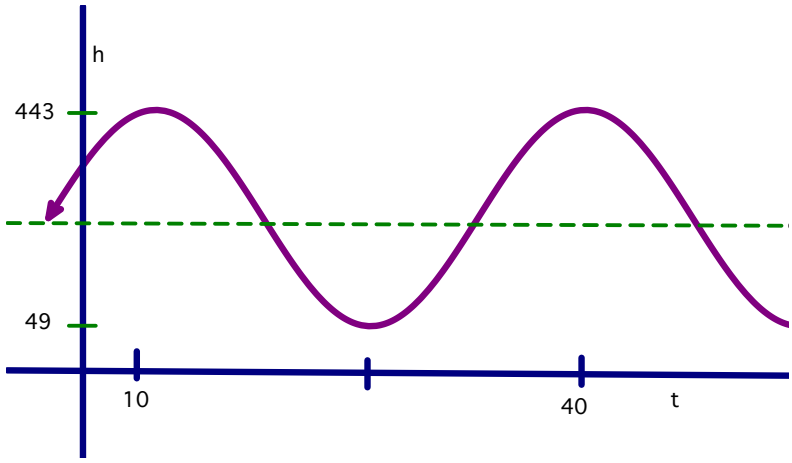
7.



2-3 Free Response Homework

1. The London Eye

a.



b. Minimum y value = 49 Maximum y value = 443

$$\Rightarrow k = \frac{443+49}{2} = 246$$

$$\Rightarrow \text{Amplitude} = 443 - 246 = 197$$

$$\text{Period } = 30$$

$$\Rightarrow B = \frac{2\pi}{30} = \frac{\pi}{15}$$

h = 10 for cosine

$$h = 246 + 197\cos\left(\frac{\pi}{15}(t-10)\right)$$

c. $h(0) = 246 + 197\cos\left(\frac{\pi}{15}(-10)\right) = 147.5\,feet$

d.

$$100 = 246 + 197\cos\left(\frac{\pi}{15}(t-10)\right)$$

$$-0.741 = \cos\left(\frac{\pi}{15}(t-10)\right)$$

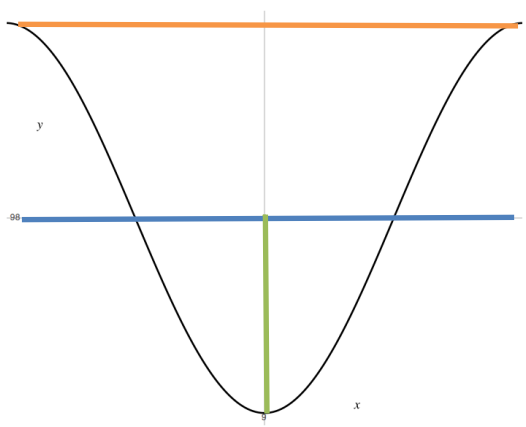
$$\frac{\pi}{15}(t-10)=\begin{cases} 2.406\pm 2\pi n \\ -2.406\pm 2\pi n \end{cases}$$

$$t-10=\begin{cases} 11.468\pm 30n \\ -11.468\pm 30n \end{cases} \quad t\in\{0.117 \text{ sec},\ 0.283 \text{ sec},\ 0.617 \text{ sec}\}$$

$$t=\begin{cases} 21.468\pm 30n \\ -1.468\pm 30n \end{cases}$$

$$t=\{21.486,\ 28.514,\ 51.486\ \textit{minutes}\}$$

3a. The Meditation Problem: $y=98+30\cos\left(\frac{\pi}{4}(t-6)\right)$



Minimum y value = 128 Maximum y value = 68

$$\Rightarrow k=\frac{128+68}{2}=98$$

$$\Rightarrow \text{Amplitude} = 128-98=98-68=30$$

$$\text{Period } =10-2=8$$

$$\Rightarrow B=\frac{2\pi}{8}=\frac{\pi}{4}$$

h = 6 for cosine (not a unique answer)

b. $y(0)=98+30\cos\left[\frac{\pi}{4}(0-6)\right]=98 \text{ cm}^3$

$$y(3) = 98 + 30\cos\left[\frac{\pi}{4}(3-6)\right] = 76.787 \text{ cm}^3$$

$$y(10) = 98 + 30\cos\left[\frac{\pi}{4}(10-6)\right] = 68 \text{ cm}^3$$

c. Highest at $t = 6,\ 14,\ 22$ seconds

$$98 + 30\cos\left[\frac{\pi}{4}(t-6)\right] = 128$$

$$\cos\left[\frac{\pi}{4}(t-6)\right] = 1$$

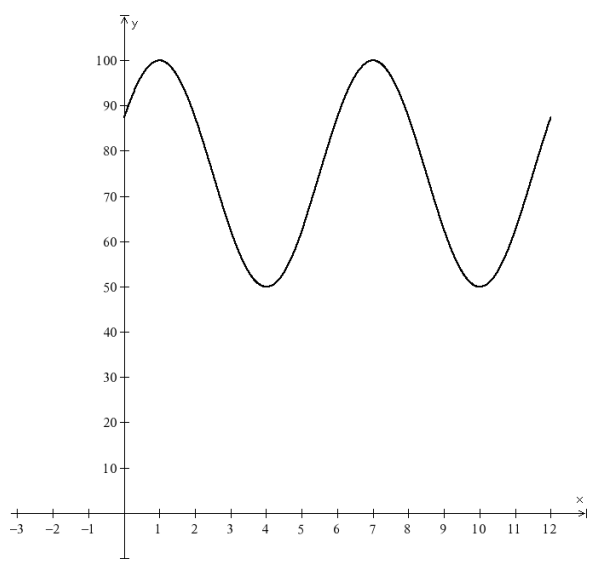
$$\frac{\pi}{4}(t-6) = \begin{cases} 0 \pm 2\pi n \\ -0 \pm 2\pi n \end{cases} = \pm 0 \pm 2\pi n$$

$$t - 6 = \{0 \pm 8n$$

$$t = \{6 \pm 8n$$

$$t \in \{6 \text{ sec},\ 14 \text{ sec},\ 22 \text{ sec}\}$$

5a.



b. Minimum y value = 50, Maximum y value = 100

$$\Rightarrow k = \frac{50+100}{2} = 75$$

$$\Rightarrow \text{Amplitude} = 100 - 75 = 75 - 50 = 25$$

$$\text{Period } = 7 - 1 = 6$$

$$\Rightarrow B = \frac{2\pi}{6} = \frac{\pi}{3}$$

$h = 1$ for cosine (not a unique answer)

$$y = 75 + 25\cos\left[\frac{\pi}{3}(t-1)\right]$$

c. $y(0) = 75 + 25\cos\left[\frac{\pi}{3}(0-1)\right] = 87.5 \textit{ students}$

d. $75 + 25\cos\left[\frac{\pi}{3}(t-1)\right] = 85$

$$\cos\left[\frac{\pi}{3}(t-1)\right] = 0.4$$

$$\frac{\pi}{3}(t-1) = \begin{cases} 1.159 \pm 2\pi n \\ -1.159 \pm 2\pi n \end{cases} = \pm 1.159 \pm 2\pi n$$

$$t - 1 = \begin{cases} 1.107 \pm 6n \\ -1.107 \pm 6n \end{cases} = \pm 1.107 \pm 6n$$

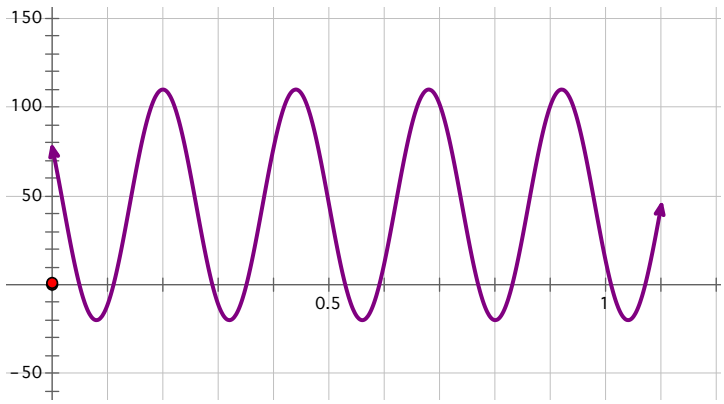
$$t - 1 = \begin{cases} 2.107 \pm 6n \\ -0.107 \pm 6n \end{cases}$$

$$t \in \{2.107 \text{ months}, 5.893 \text{ months}\}$$

7. The Trampoline Problem

If you decide the trampoline surface is zero:

a.



b. Minimum H value = -20
Maximum H value = 110

$$\Rightarrow k = \frac{110+(-20)}{2} = 45$$

$$\Rightarrow \text{Amplitude} = 110 - 45 = 65$$

$$\text{Period } = 0.24$$

$$\Rightarrow B = \frac{2\pi}{0.24} = \frac{25\pi}{3}$$

$$h = 0.2$$

$$h(t) = 45 + 65\cos\left(\frac{25\pi}{3}(t-0.2)\right)$$

c. One meter above the ground is 55 cm above the trampoline surface. So,

$$55 = 45 + 65\cos\left(\frac{25\pi}{3}(t-0.2)\right)$$

$$\cos\left(\frac{25\pi}{3}(t-0.2)\right) = \frac{10}{65}$$

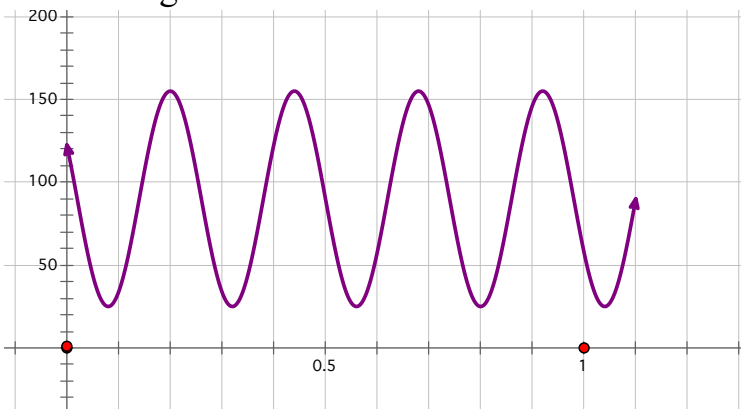
$$\frac{25\pi}{3}(t-0.2) = \pm 1.416 \pm 2\pi n$$

$$t - 0.2 = \pm .054 \pm .24n$$

$$t = \begin{cases} 0.254 \pm .24n \\ 0.146 \pm .24n \end{cases}$$

$$t = 0.019,\ 0.222, 0.259,\ 0.462 \text{ secs}$$

If you decide the ground is zero:

a. 

b. Minimum H value = 25
Maximum H value = 155

$$\Rightarrow k = \frac{155+25}{2} = 90$$

$$\Rightarrow \text{Amplitude} = 155 - 95 = 65$$

$$\text{Period } = 0.24$$

$$\Rightarrow B = \frac{2\pi}{0.24} = \frac{25\pi}{3}$$

$$h = 0.2$$

$$h(t) = 90 + 65\cos\left(\frac{25\pi}{3}(t-0.2)\right)$$

c. One meter above the ground is 100 cm. So, $h = 100\,\text{cm} \rightarrow$

$$100 = 90 + 65\cos\left(\frac{25\pi}{3}(t-0.2)\right)$$

$$\cos\left(\frac{25\pi}{3}(t-0.2)\right) = \frac{10}{65}$$

$$\frac{25\pi}{3}(t-0.2) = \pm 1.416 \pm 2\pi n$$

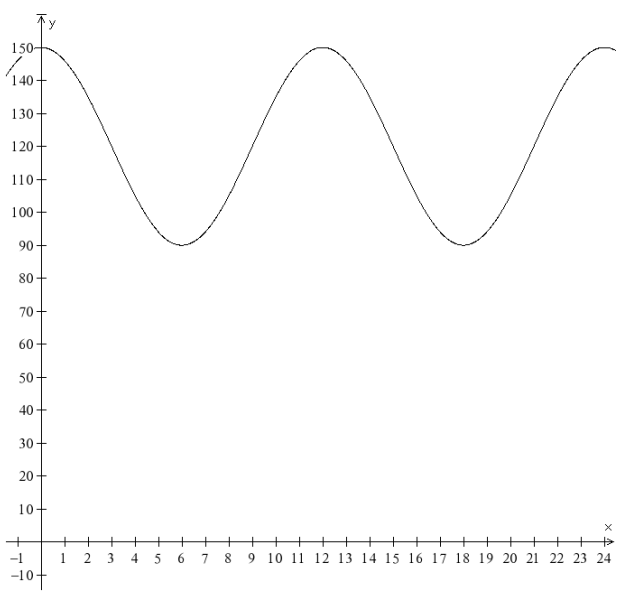
$$t - 0.2 = \pm .054 \pm .24n$$

$$t = \begin{cases} 0.254 \pm .24n \\ 0.146 \pm .24n \end{cases}$$

$$t = 0.019,\ 0.222, 0.259,\ 0.462 \text{ secs}$$

9. The Hospital Patient Problem

a.



b. Minimum P value = 90
Maximum P value = 150

$$\Rightarrow k = \frac{90+150}{2} = 120$$

$$\Rightarrow \text{Amplitude} = 150 - 120 = 120 - 90 = 30$$

$$\text{Period } = 9\text{am} - 9\text{pm} = 12 \text{ hours}$$

$$\Rightarrow B = \frac{2\pi}{12} = \frac{\pi}{6}$$

$h = 12$ for cosine (not a unique answer)

$h = 9$ for sine (not a unique answer)

$$P(t) = 120 + 30\sin\left[\frac{\pi}{6}(t-9)\right] \; or \; P(t) = 120 + 30\cos\left[\frac{\pi}{6}t\right]$$

c. $$P(13.25) = 120 + 30\sin\left[\frac{\pi}{6}(13.25-9)\right] = 143.801 \text{ mmHg}$$

d. $$120 + 30\sin\left[\frac{\pi}{6}(t-9)\right] = 130$$

$$\sin\left[\frac{\pi}{6}(t-9)\right] = 1/3$$

$$\frac{\pi}{6}(t-9) = \begin{cases} 0.340 \pm 2\pi n \\ 2.802 \pm 2\pi n \end{cases}$$

$$t - 9 = \begin{cases} 0.649 \pm 12n \\ 5.351 \pm 12n \end{cases}$$

$$t = \begin{cases} 9.649 \pm 12n \\ 14.351 \pm 12n \end{cases}$$

$$t \in \{2.351,\ 9.649,\ 14.351\}$$

$0.351 \bullet 60 = 21.06$

$0.649 \bullet 60 = 38.94$

2:21 am, 9:39 am, 2:21 pm

11. The Whale Problem

Minimum y value = –7 Maximum y value = 1

$$k = \frac{-7+1}{2} = -3$$

Amplitude = $-7-(-3) = -4$

Period $= 80 \Rightarrow B = \frac{2\pi}{80} = \frac{\pi}{40}$

$h = 44$ for a positive cosine (not a unique answer)

$$d = -3 + 4\cos\left(\frac{\pi}{40}(t-44)\right)$$

$$0 = -3 + 4\cos\left(\frac{\pi}{40}(t-44)\right)$$

$$\frac{\pi}{40}(t-44) = \cos^{-1}\left(\frac{3}{4}\right) = \begin{cases} 0.773 \pm 2\pi n \\ -0.773 \pm 2\pi n \end{cases}$$

$$t-44 = \begin{cases} 9.202 \pm 80n \\ -9.202 \pm 80n \end{cases}$$

$$t = \begin{cases} 53.202 \pm 80n \\ 34.798 \pm 80n \end{cases}$$

$$t = 114.798 \text{ sec}$$

13. Game 2, Game 18, and Game 56

High value of .425 on Day 10 and low value of 1.75 on Day 64, so

$$\Rightarrow k = \frac{.425 + .175}{2} = .300$$

$$\Rightarrow \text{Amplitude} = .425 - .300 = .125$$

$$\text{Period } = 54$$

$$\Rightarrow B = \frac{2\pi}{54} = \frac{\pi}{27}$$

$$h = 10$$

$$B(t) = .300 + .125\cos\left[\frac{\pi}{27}(t-10)\right]$$

$$.374 = .300 + .125\cos\left[\frac{\pi}{27}(t-10)\right]$$

$$.6 = \cos\left[\frac{\pi}{27}(t-10)\right]$$

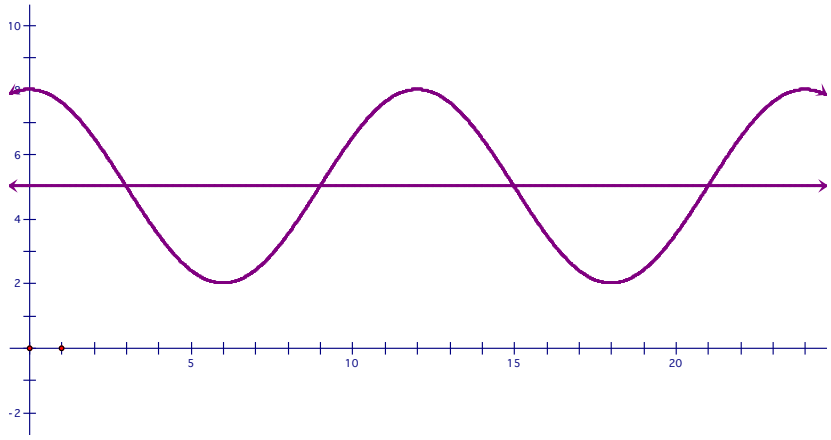
$$\frac{\pi}{27}(t-10) = \cos^{-1}(.6) = \begin{cases} .927 \pm 2\pi n \\ -.927 \pm 2\pi n \end{cases}$$

$$t - 10 = \begin{cases} 7.970 \pm 54n \\ -7.970 \pm 54n \end{cases}$$

$$t = \begin{cases} 17.970 \pm 54n \\ 2.030 \pm 54n \end{cases}$$

First three times: Game 2, Game 18, and Game 56

15.



$$y = 5 + 3\cos\left[\frac{\pi}{6}t\right]$$

$$3 = 5 + 3\cos\left[\frac{\pi}{6}t\right]$$

$$3 = 5 + 3\cos\left[\frac{\pi}{6}t\right]$$

$$-\frac{2}{3} = \cos\left[\frac{\pi}{6}t\right]$$

$$\left\{\begin{matrix} 2.301 \pm 2\pi n \\ -2.301 \pm 2\pi n \end{matrix}\right\} = \frac{\pi}{6}t$$

$$\left\{\begin{matrix} 4.394 \pm 12n \\ -4.394 \pm 12n \end{matrix}\right\} = t$$

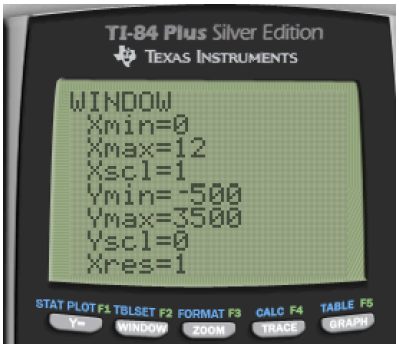
$$t = 4.394,\ 7.606,\ 16.394,\ 19.606$$

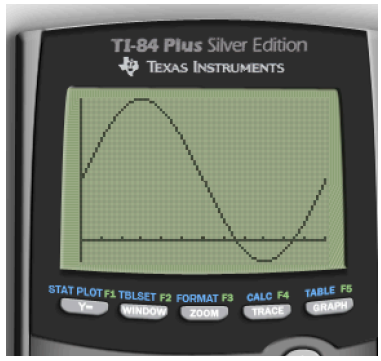
The times of day are 4:24am, 7:36am, 4:24pm, and 7:36pm.

<u>2-3 Multiple Choice Homework</u>

1. D $\qquad \dfrac{25+16}{2} = 20.5$

3. B The graph goes below the x-axis after 7 months.

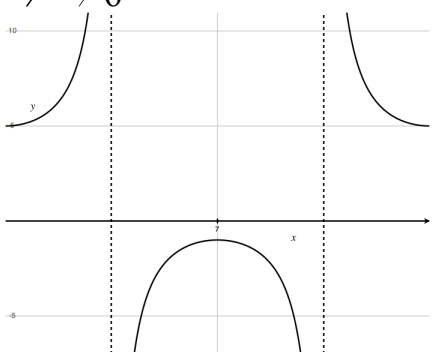




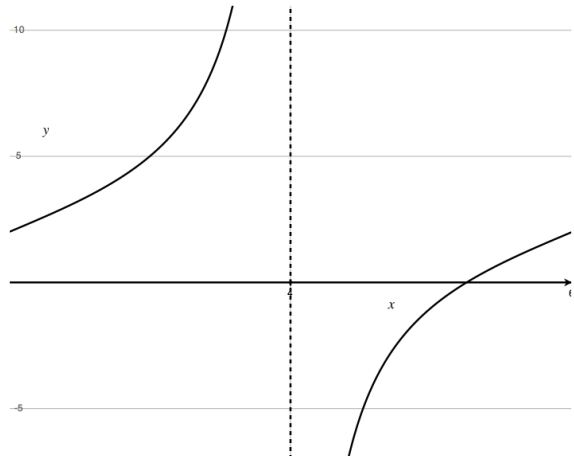
5. D. $period = 4\sec;\ Number\ of\ rotations = 2hr \cdot 3600\frac{\sec}{hr} \div 4\sec = 1800$

<u>2-4 Free Response Homework</u>

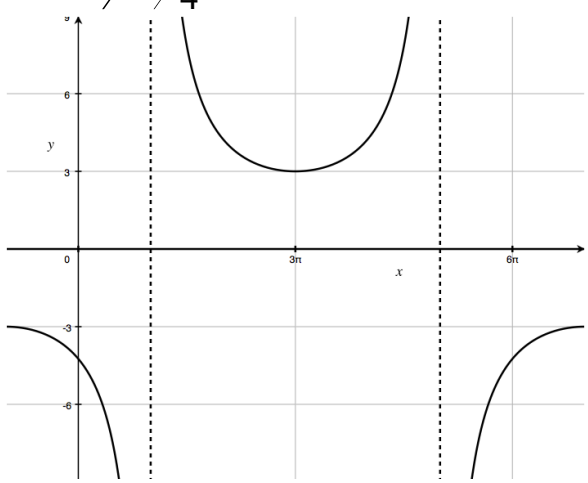
1. $A = 3$ period $= \dfrac{2\pi}{\frac{\pi}{6}} = 12$ $k = 2$ $h = 1$



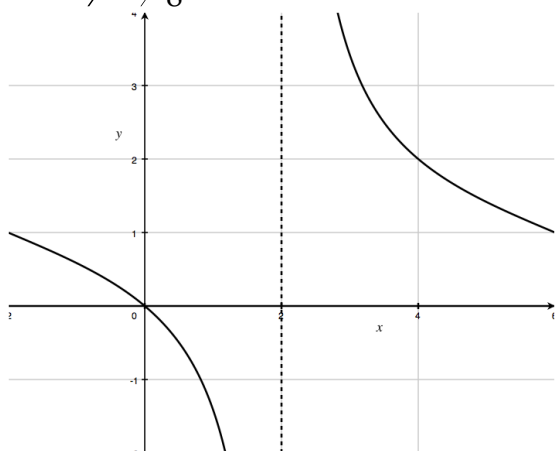
3. $A = 3$ period = $\dfrac{\pi}{\frac{\pi}{4}} = 4$ $k = 2$ $h = 2$



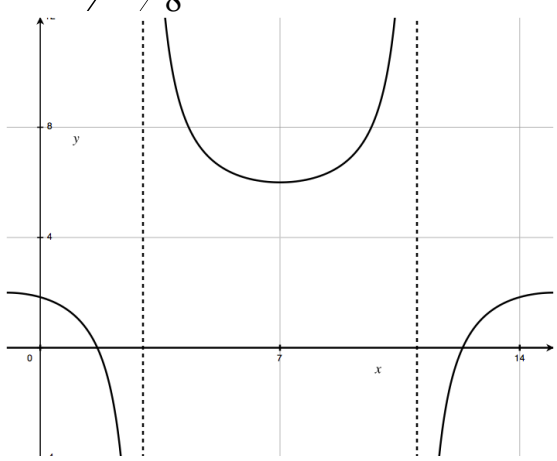
5. $A = 3$ period = $\dfrac{2\pi}{\frac{1}{4}} = 8\pi$ $k = 0$ $h = -\pi$



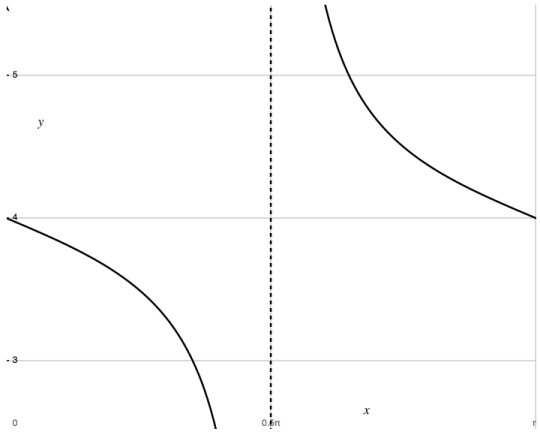
7. $A = 1$ period = $\dfrac{\pi}{\frac{\pi}{8}} = 8$ $k = 1$ $h = -2$



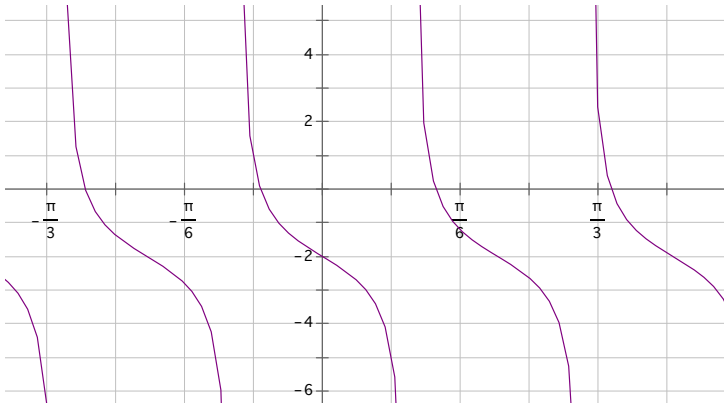
9. $A = -2$ period = $\dfrac{2\pi}{\frac{\pi}{8}} = 16$ $k = 4$ $h = -1$



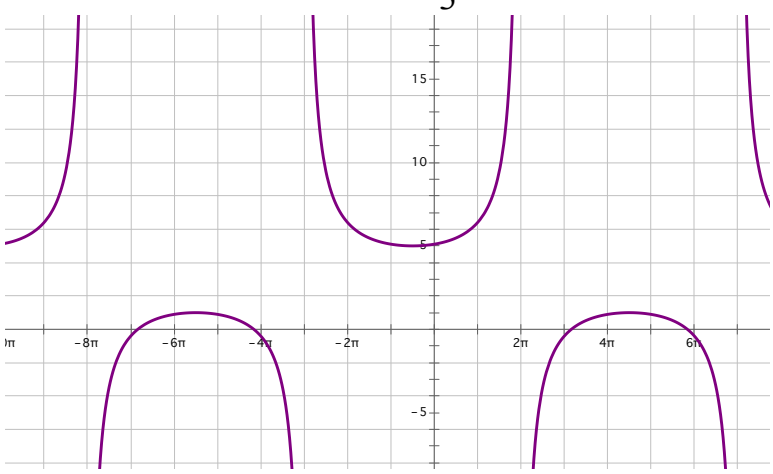
11. $A = -\dfrac{1}{2}$ period = $\dfrac{\pi}{1} = \pi$ $k = 4$ $h = 0$



13. $A = 1$ Period = $\frac{4}{3}$ $k = -2$ $h = 1$



15. $A = 2$ Period = $\frac{4}{3}$ $k = 3$ $h = -3\pi$



17. $y = 1 - \frac{1}{2}\tan\left[\frac{\pi}{8}(t-1)\right]$ and $y = 1 + \frac{1}{2}\cot\left[\frac{\pi}{8}(t+1)\right]$

midpoint y-value = 1=k

or quarter-point y-values are $\frac{3}{2}$ and $\frac{1}{2}$ $\therefore k = 1$

or quarter-point y-values are $\frac{3}{2}$ and $\frac{1}{2}$ $\therefore A = \frac{3/2 - 1/2}{2} = \frac{1}{2}$

Asymptote values = 5 and $-3 \Rightarrow$ Period $= 5 - (-3) = 8 \Rightarrow B = \frac{\pi}{8}$

$h = -3$ for cot (not a unique answer)

$h = 1$ for tan, but the curve is upside-down, so A= $-\frac{1}{2}$

19. $y = 2 + 2\sec\left[\frac{\pi}{6}(t-1)\right]$ and $y = 2 + 2\csc\left[\frac{\pi}{6}(t+2)\right]$

Minimum y value = 0 Maximum y value = -4

$\Rightarrow k = \frac{0-4}{2} = -2$

$\Rightarrow$ Amplitude = $0 - (-2) = 2$

Period $= 4 - (-8) = 12 \Rightarrow B = \frac{2\pi}{12} = \frac{\pi}{6}$

$h = 1$ for sec (not a unique answer)

$h = -2$ for csc (not a unique answer)

<u>2-4 Multiple Choice Homework</u>

1. B

$x = \tan^{-1}(3) = 1.249$

$\sqrt{\csc x} = \sqrt{\csc 1.249} = \sqrt{\frac{1}{\sin 1.249}} = 1.027$

3. A

$y = \frac{\sin x}{1 + \cos x} = \tan\frac{1}{2}x \Rightarrow p = \frac{\pi}{B} = \frac{\pi}{1/2} = 2\pi$

5. D

$$5-\sin\left(x+\frac{\pi}{6}\right)=6$$

$$\sin\left(x+\frac{\pi}{6}\right)=-1$$

$$x+\frac{\pi}{6}=\begin{cases}\frac{3\pi}{2}\pm 2\pi n\\ -\frac{\pi}{2}\pm 2\pi n\end{cases}=\left\{\frac{3\pi}{2}\pm 2\pi n\right.$$

$$x=\left\{\frac{4\pi}{3}\pm 2\pi n\right.$$

$$x=\frac{4\pi}{3}=4.19$$

Trigonometric Functions Practice Test Solutions

Multiple Choice

1. A

$$\text{period} = 180° = \pi \Rightarrow B = {2\pi}/{\pi} = 2$$

$$k = 3$$

$$A = 1$$

$$\Rightarrow y = 3 + 1 \bullet \sin\left[2\left(x - 0\right)\right]$$

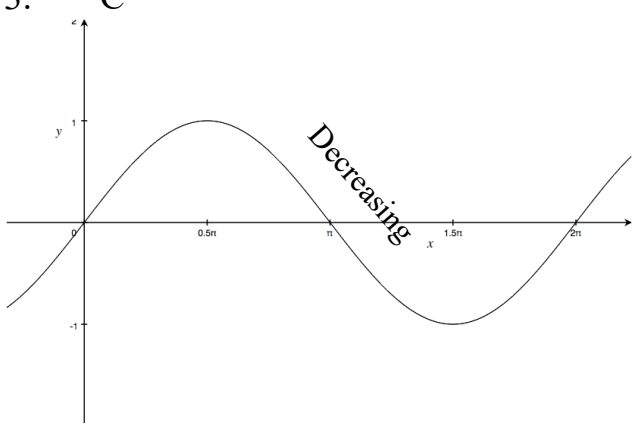
$$\therefore y = \sin 2x + 3$$

2. B

$$y = abs\left(\frac{\sqrt{2}}{\sin\left(3\left(0 + \frac{\pi}{5}\right)\right)}\right) = abs\left(\frac{\sqrt{2}}{\sin\frac{3\pi}{5}}\right) \approx 1.487$$

Note: abs is found on calculator
by Math $\rightarrow$ Num $\rightarrow$ #1

3. C



4. B $\quad$ Amplitude $= \dfrac{\text{Max} - \text{Min}}{2} = \dfrac{68-44}{2} = 12$

5. A $\quad$ Period $= \dfrac{3.6-1.2}{2} = 1.2$

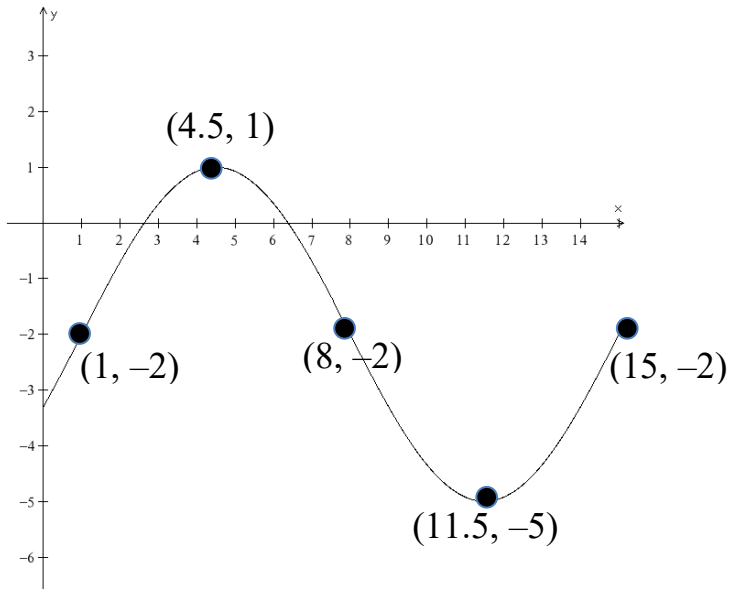
<u>Free Response</u>

1. $A = 3$

   Period $= \dfrac{2\pi}{\pi/7} = 14\pi$

   $k = -2$

   $h = 1$

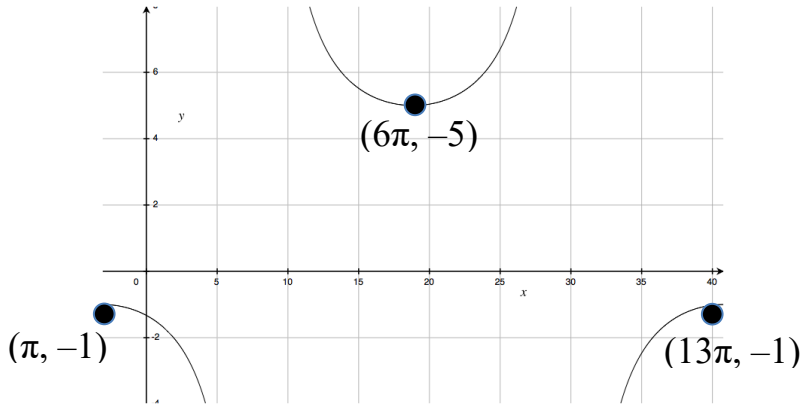


2. $A = 3$

   Period $= \dfrac{2\pi}{1/7} = 14\pi$

   $k = 2$

   $h = -\pi$

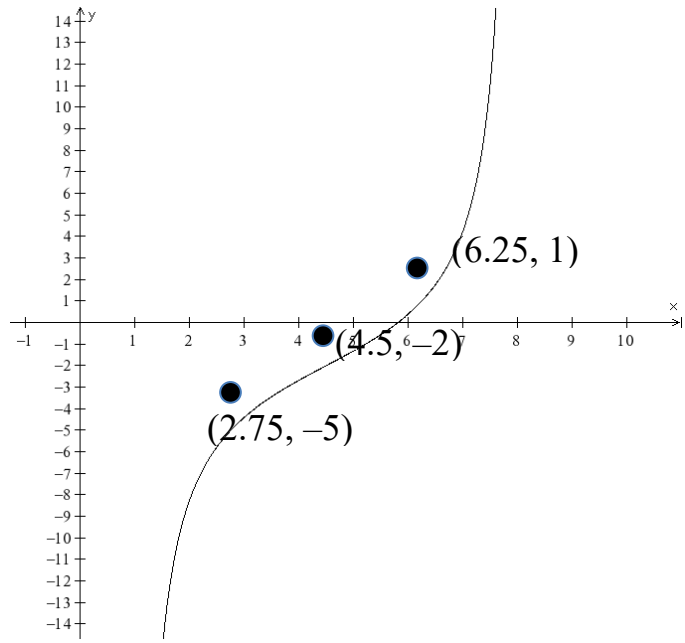


3. $A = 3$

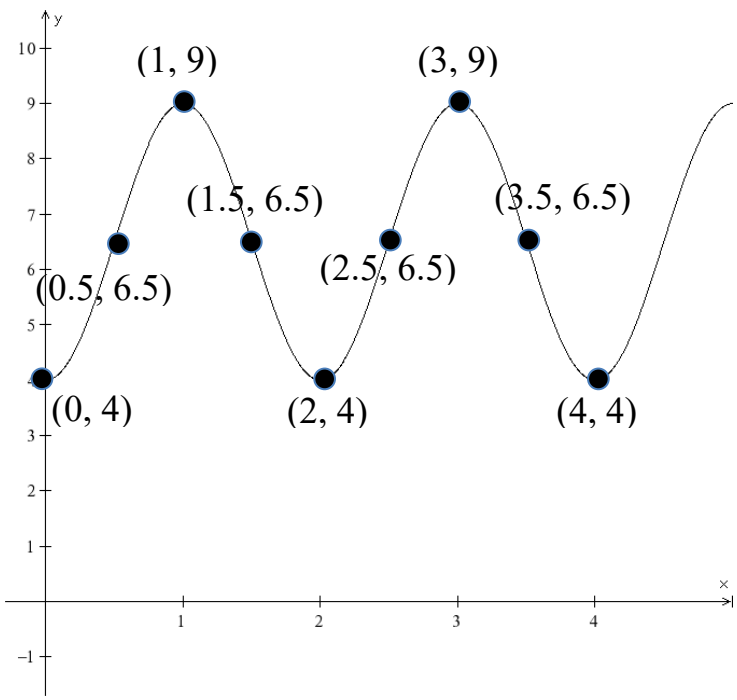
period = $\pi \Big/ \frac{\pi}{7} = 7$

$k = -2$

$h = 1$



4a.



b. $h = 6.5 - 2.5\cos\left[\pi\left(t - 0\right)\right]$

$A = 2.5$

period = 2

$\Rightarrow B = 2\pi \Big/ 2 = \pi$

$k = 6.5$

$h = 0$

c. $h = 6.5 - 2.5\cos\left[\pi\left(7-0\right)\right] = 9 \text{ feet}$

d. $6.5 - 2.5\cos\left[\pi\left(t-0\right)\right] = 6$

$$\cos\left[\pi\left(t-0\right)\right] = 0.2$$

$$\pi\left(t-0\right) = \begin{cases} 1.369 \pm 2\pi n \\ -1.369 \pm 2\pi n \end{cases} = \pm 1.369 \pm 2\pi n$$

$$t - 0 = \left\{\pm 0.436 \pm 2n\right.$$

$$t = \left\{\pm 0.436 \pm 2n\right.$$

$$t \in \left\{0.436 \text{ sec},\ 1.537 \text{ sec},\ 2.263 \text{ sec},\ 3.537 \text{ sec}\right\}$$